## 1994 Paper 6 Question 10

## Foundations of Functional Programming

Describe the operation of a graph reducer and its treatment of the combinators $\mathbf{K}$, $\mathbf{S}, \mathbf{Y}$, if (for conditional expressions) and mult (integer multiplication). [6 marks]

Describe the operation of the SECD machine, including its treatment of recursive functions.

Exhibit an infinite family $\Phi_{n}$ of distinct fixed-point combinators. Justify your answer by showing that $\Phi_{n} \rightarrow F\left(\Phi_{n} F\right)$ for all non-negative integers $n$ and $\lambda$-terms $F$. You must also show that $\Phi_{m} \neq \Phi_{n}$ for $m \neq n$, quoting standard results about the $\lambda$-calculus if necessary.

