1994 Paper 5 Question 11

Computation Theory

The Halting Problem for register machines is unsolvable. State, without proof, a precise form of this result. [3 marks]

Let the computation by program p on data d be represented by the natural number k that codes the pair (p, d). By considering the set H(k) of the HALTing computations represented by codes $k' \leq k$, show that there is an increasing total function h(k) which grows too fast to be computable. [6 marks]

Given $h : \mathbb{N} \to \mathbb{N}$ with the above property

let
$$f(k) = h(k) + k$$

and $g(x) = \sup\{k : f(k) \le x\}.$

Then $f: \mathbb{N} \to \mathbb{N}$ is strictly increasing, and $g: \mathbb{N} \to \mathbb{N}$ satisfies

g(f(k)) = k, g(x) < k for all x < f(k).

Show that g grows too slowly to be computable in the following sense... given $G: \mathbb{N} \to \mathbb{N}$ such that

(a) $\{G(n) : n \in \mathbb{N}\}$ is unbounded

(b) $G(n) \leq g(n)$ for all $n \in \mathbb{N}$

then G(n) is not computable.

[11 marks]