COMPUTER SCIENCE TRIPOS Part II

Tuesday 31 May 1994 1.30 to 4.30

Paper 7

Answer **five** *questions.*
Submit the answers in five **separate** *bundles each with its own cover sheet.*
Write on **one** *side of the paper only.*

1 Developments in Technology

Semiconductor technology has made impressive progress in recent years. Write short notes about

(*a*) the factors contributing to this progress [10 marks]

(*b*) the prospects for its continuation for, say, a further 10 years [5 marks]

Are there signs that devices in the near future could/should exploit different principles from those of today? [5 marks]

2 Additional Topics I and II

Define a Hamming Code and show how it can correct a single error. [3+3 marks]

Show how, by using an extra parity bit, it can detect a double error. [3 marks]

Give an example of a convolutional code. [5 marks]

Given a BCH code (1023, 983), which errors can it correct? [2 marks]

Give an example where a run length code is better than a Huffman Code. [4 marks]

3 Computer Systems Modelling

Consider a system with two independent server devices, which have FIFO queues, and two distinct customers. All service times are exponentially distributed but the service time distributions for the customers and devices differ; that is, there are four distinct service rates, one for each customer at each device.

(*a*) Draw the state diagram of a Markov chain representing the system. [8 marks]

(*b*) If all the service rates are equal then the system corresponds to our notion of a balanced system. Show that under these conditions the utilisation of each device is given by

$$U = \frac{N}{N + K - 1}$$

where N is the number of customers and K is the number of devices. [6 marks]

(*c*) Show that this expression also holds in a system with three indistinguishable customers and two devices with identically exponentially distributed service times. [6 marks]

4 Digital Communication II

Two Ethernets can be interconnected using a *bridge*, which forwards packets between the networks at the OSI Datalink layer. Describe the operation of an Ethernet bridge. [7 marks]

In a network composed of several bridged Ethernets joined in an arbitrary topology, describe how the problem posed by multiple alternative paths is solved. [8 marks]

As the number of connected systems gets large, how can the performance of the packet forwarding in such a network be improved by use of suitable hardware and/or software? [5 marks]

5 Philosophy

Discuss the relation between mind and body. [20 marks]

6 Algebraic Manipulation

Write short notes on the use of *remainder sequences* in each of the following contexts:

(a) finding the GCD of two polynomials

(b) eliminating a variable between two polynomial equations

(c) counting and isolating the roots of a univariate polynomial equation

(d) solving the equation $Ax + By = C$ to find x and y (where A, B, C, x and y are integers)

[20 marks]

7 Natural Language Processing

Discuss some of the methods used by current NLP systems to carry out

(a) morphological analysis [6 marks]

(b) parsing [6 marks]

(c) semantic interpretation [8 marks]

Illustrate your answer by suggesting appropriate representations, and how to construct them, for the following sentence:

Every borrower repaid his loan from Cambank.

8 Computational Neuroscience

Explain the following methods for updating the weights in associative networks:

(a) the Hebbian rule [5 marks]

(b) the Delta Rule (or LMS or Widrow–Hoff rule) [5 marks]

(c) the rule used by Kohonen feature maps [5 marks]

What are the advantages and disadvantages of each? [5 marks]

9 Optimising Compilers

Consider the following ANSI C routine:

```
 struct List { int hd; struct List *tl; };
 struct List *readlist()
 {   int i;
     struct List *p, *q, *t;
l1:  p = 0;
l2:  while (scanf("%d", &i) == 1)
     {
l3:      t = malloc(sizeof(List));
         if (t == 0) abort();
         t->hd = i;
l4:      t->tl = 0;
         if (p == 0)
             p = q = t;
         else
             q->tl = t, q = t;
     }
l5:  return p;
 }
```

Summarise its function. [4 marks]

Sketch an algorithm which computes liveness of variables. [6 marks]

Calculate the sets of (local) variables which are live at `l1`, `l2`, `l3`, `l4` and `l5`. [4 marks]

Do any of `p`, `q`, `t` and `i` have dataflow anomalies? Briefly justify your answer indicating which anomalies may represent potential faults. [6 marks]

10 Numerical Analysis II

Explain the terms *Riemann integral* and *Riemann sum.* [3 marks]

Let $\mathbf{R}$ be a quadrature rule that integrates constants exactly. If a function f is bounded and Riemann-integrable over the interval $[a, b]$ then prove that

$$\lim_{n\to\infty} (n \times \mathbf{R})f = \int_a^b f(x)dx.$$ [6 marks]

Consider two quadrature rules for the interval $[-\lambda, \lambda]$:

$$\mathbf{S}f = \frac{\lambda}{3}\{f(-\lambda) + 4f(0) + f(\lambda)\} - \frac{\lambda^5}{90} f^{(4)}(\xi)$$
$$\mathbf{T}f = \lambda\{f(-\lambda) + f(\lambda)\} - \frac{2}{3}\lambda^3 f''(\zeta)$$

If $\mathbf{S}$ were used in the composite form $(n \times \mathbf{S})f$, what order of convergence would you expect? [2 marks]

Suppose the rule

$$\frac{1}{3}\{F(-1,-1) + 4F(-1,0) + F(-1,1) + F(1,-1) + 4F(1,0) + F(1,1)\}$$

is applied to

$$\int_{-1}^{1}\int_{-1}^{1} F(x,y)\ dxdy.$$

Describe the 2-variable polynomials that are integrated exactly by this rule. [6 marks]

Why is the product form of $\mathbf{S}f$ unsuitable for integrating over a hypercube in 20 dimensions? Name a better method for 20 dimensions on a sequential machine, given that high accuracy is not required. [3 marks]

11 Proving Programs Correct

Describe the rôle of first-order predicate calculus in Floyd–Hoare logic. [5 marks]

Give an outline of a deductive system for predicate calculus, covering at least the following topics:

(*a*) terms, formulae and sequents [5 marks]

(*b*) introduction *versus* elimination rules [5 marks]

(*c*) side conditions and variable capture [5 marks]

Illustrate your explanations with examples.

12 Computational Number Theory

Describe how the set of composite numbers can be recognised in Random Polynomial time. [12 marks]

Describe how the set of prime numbers can be recognised in Non-deterministic Polynomial time. [8 marks]

13 Semantics of Programming Languages

Dijkstra proposed the language of guarded commands with the following syntax. Commands take the form

$$c \quad ::= \quad \mathsf{skip} \quad | \quad \mathsf{abort} \quad | \quad X := e \quad | \quad c; c \quad | \quad \mathsf{if}\ gc\ \mathsf{fi} \quad | \quad \mathsf{do}\ gc\ \mathsf{od}$$

where e is an arithmetic expression and gc stands for a guarded command of the form

$$b_1 \to c_1 \,[\!]\, b_2 \to c_2$$

for boolean expressions b_1 and b_2, called *guards*, and commands c_1 and c_2. Execution of the command skip does not result in a change of state. Following Dijkstra's intentions, if no guard evaluates to true at a state, then the guarded command is said to fail, in which case, the guarded command does not yield a final state. Otherwise, the guarded command executes as one of the commands c_i whose associated guard b_i evaluates to true. The execution of the command abort does not yield a final state from any initial state. The command $\mathsf{if}\ gc\ \mathsf{fi}$ executes as the guarded command gc, if gc does not fail, otherwise, it acts like abort. The command $\mathsf{do}\ gc\ \mathsf{od}$ executes repeatedly as the guarded command gc, while gc continues not to fail, and terminates when gc fails.

(a) Assume that boolean and arithmetic expressions have no side effects and always terminate, and that the rules for their evaluation are given. Write down a collection of rules for an inductively defined evaluation relation of the form

$$c,\ S \ \Rightarrow\ S'$$

whose sense is "starting from the initial state S, the evaluation of the command c terminates at the final state S'." [10 marks]

(b) Give the commands in Dijkstra's guarded language which simulate the standard imperative programming commands

$$\mathsf{if}\ b\ \mathsf{then}\ c_1\ \mathsf{else}\ c_2 \qquad \text{and} \qquad \mathsf{while}\ b\ \mathsf{do}\ c$$

respectively. You may assume that if b is a boolean expression, then so is $\neg b$, the negation of b. [2 marks]

(c) Give an appropriate definition of *semantic equivalence* of commands with respect to the evaluation relation defined in (a). Prove that for any boolean expression b and any command c, the command

$$\mathsf{do}\ b \to c \,[\!]\, b \to c\ \mathsf{od}$$

is semantically equivalent to the command

$$\mathsf{if}\ b \to (c; \mathsf{do}\ b \to c \,[\!]\, b \to c\ \mathsf{od}) \,[\!]\, \neg b \to \mathsf{skip}\ \mathsf{fi}$$ [8 marks]

14 Concurrency

Define what is meant by *observational equivalence* of CCS agents. [5 marks]

A transmitter T, transmission medium M, and receiver R are modelled by CCS agents with the following definitions:

$$
\begin{aligned}
T &\stackrel{\text{def}}{=} in.\overline{i}.T' \\
T' &\stackrel{\text{def}}{=} r.\overline{i}.T' + a.T \\
M &\stackrel{\text{def}}{=} i.M' \\
M' &\stackrel{\text{def}}{=} \overline{o}.M + \tau.\overline{r}.M \\
R &\stackrel{\text{def}}{=} o.\overline{out}.\overline{a}.R
\end{aligned}
$$

M is an unreliable medium: having received an input message from T (action i) it either outputs the message to R (action $\overline{o}$), or loses it (represented by the τ action) and then sends a request for retransmission (action $\overline{r}$). If R does receive the message, after broadcasting it (action $\overline{out}$) it sends an acknowledgement directly to T (action $\overline{a}$).

Calculate the transition graph of $(T|M|R) \setminus \{i, o, r, a\}$ and hence show that this agent is observationally equivalent to a simple buffer B with definition

$$B \stackrel{\text{def}}{=} in.\overline{out}.B$$ [10 marks]

Are $(T|M|R) \setminus \{i, o, r, a\}$ and B observationally congruent? [3 marks]

Do the two agents have the same behaviour with respect to *divergence*, that is, the ability to perform a series of actions ending in an infinite sequence of τ-actions?
[2 marks]