1993 Paper 9 Question 1

Digital Signal Processing

Describe in detail the use of the bilinear transformation

$$s \to \frac{2}{T} \frac{z-1}{z+1}$$

for the design of digital filters from analogue filters. Discuss the advantages and disadvantages of the method. [8 marks]

It is required to design a second order maximally flat digital low-pass filter having a 3dB cut-off frequency of $0.2 f_s$ where f_s is the sampling frequency. The design is to be based on the analogue Butterworth filter defined by:

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\dfrac{\omega}{\omega_c}\right)^{2N}}$$

where N is the filter order and ω_c is the 3dB cut-off frequency. It can be shown that the s-domain transfer function for the second order Butterworth filter is given by:

$$H(s) = \frac{1}{1 + \sqrt{2}\left(\dfrac{s}{\omega_c}\right) + \left(\dfrac{s}{\omega_c}\right)^2}$$

Calculate the coefficient values for the corresponding digital low-pass filter and draw its block diagram. [12 marks]