In a meteorological experiment the annual rainfall in millimetres is recorded for Aberdeen, Bangor, Canterbury and Dublin: readings are taken in each of the years 1981–83. The data are represented as a set of triples (r, t, y), where rainfall $r \in \mathbb{N}$ is a natural number of millimetres, the town $t \in T$ is identified by a letter A–D, and the year $y \in Y$ by a digit 1–3. Show how to identify the data with subsets of each of $(\mathbb{N} \times T) \times Y$, $(T \times Y) \times \mathbb{N}$ and $(Y \times \mathbb{N}) \times T$. Which of these relations **either** *must* **or** *may* define a (partial) function from $(\mathbb{N} \times T) \to Y$, $(T \times Y) \to \mathbb{N}$, $(Y \times \mathbb{N}) \to T$ respectively?

(*a*) Taking one which *must* be a function $(P \times Q) \to R$ say, show how to convert it to a function whose domain is P and whose range is the set of functions $Q \to R$.

(*b*) Assume that one of the other relations, which *may* define a partial function, does so. Show how to convert it to a partial function whose range is a set of partial functions.

Illustrate your answer with suitably chosen example data.