1993 Paper 11 Question 10

Numerical Analysis I

What is meant by the term *loss of significance*? What is the essential difference between the terms *condition* and *stability* in numerical analysis? Define the term *machine epsilon* and explain why it is an important parameter. [6 marks]

Use the recurrence formula

$$\cos[(k+1)\theta] = 2\cos\theta\cos[k\theta] - \cos[(k-1)\theta]$$

with starting values $\cos 0 = 1$, $\cos\theta = \frac{1}{\sqrt{2}} + \varepsilon$ to evaluate $\cos 2\theta$ and show that loss of significance occurs. [4 marks]

Evaluate $\cos 3\theta$ and $\cos 4\theta$, ignoring terms $O(\varepsilon^3)$. On this evidence, comment on the stability of the formula. [8 marks]

Is the computed value of $\cos 2\theta$ acceptable? Explain your answer. [2 marks]