## COMPUTER SCIENCE TRIPOS Part Iв

Tuesday 1 June 19931.30 to 4.30

Paper 4
Answer five questions.
Submit the answers in five separate bundles each with its own cover sheet.
Write on one side of the paper only.

## 1 Further Modula-3

Describe the facilities in Modula-3 for defining generic interfaces and modules.
[5 marks]
Illustrate your answer by sketching a generic interface and implementation for a Bag, that is, a collection of items of the same type (possibly including duplicates), implemented as an opaque object with insert and remove methods. [10 marks]

The default Bag should be implemented as a stack, using a last-in first-out policy. Describe briefly alternative implementations providing specialisations of Bag to give a Queue (first-in first-out) and Set (ignoring duplicates).

## 2 Common Lisp

You are asked by your manager to write a Lisp macro, itercall. Evaluating (itercall $F E$ ) evaluates $E$, which is expected to yield a non-negative integer $n$. It then executes the function calls ( $F 1$ ), ..., ( $F n$ ) in succession, and returns nil.
(a) Your first version of the macro expands to a loop, which uses the symbol i as an index variable and the symbol n to store the initial value of $E$. Present the code for this version.
(b) Your manager complains that the function

```
(defun test1 (i) (itercall (lambda (x) (print (cons x i))) 10))
```

does not work as expected. Explain the problem and suggest how to fix it by modifying the macro.
(c) Your manager requests a final modification: (itercall $F E$ ) should generate straight-line code instead of a loop provided $E$ is an integer constant less than twenty. Present the code for this version. Will it run faster than the previous versions?

Note: (integerp $x$ ) tests whether $x$ is an integer. Each time (gensym) is called, it returns a new symbol not previously used in the Lisp system.

## 3 Software Engineering

Give a brief description of the main constructs used in a VDM specification.
[7 marks]
Discuss to what extent the notation used in VDM is significantly different from that used in a conventional programming language.

Use VDM to specify a function that will find the difference between the largest and the smallest values held in an integer array.
[7 marks]

## 4 Prolog

The following Prolog clauses define the procedure named reverse. The goal reverse ( $\mathrm{X}, \mathrm{Y}$ ) succeeds for the list X , instantiating Y to the reverse of the list X . For example, evaluating the goal reverse ( $[\mathrm{a}, \mathrm{b}, \mathrm{c}], \mathrm{Q}$ ) instantiates Q to $[\mathrm{c}, \mathrm{b}, \mathrm{a}$ ].

```
reverse(X,Y) :- rev(X,[],Y).
rev([],L,L).
rev([H|T],R,Y) :- rev(T,[H|R],Y).
```

Explain how this procedure works, using a small example.
What is the outcome of the goal reverse( $L,[a, b, c]$ )? Explain your answer carefully.

## 5 Programming Language Compilation

Give a brief description of the main features of Lex and Yacc.
Illustrate their use by outlining how you would construct a parser for expressions composed of identifiers, integers, function calls and the operators $*, /,+$ and - .
[10 marks]

## 6 UNIX Case Study

Show how race conditions can arise:
(a) among processes over access to shared data
(b) between processes and interrupt-driven routines

Discuss why the UNIX kernel cannot be run on a shared-memory multiprocessor.

Outline how the UNIX kernel could be modified to run on a shared-memory multiprocessor.

Describe briefly an alternative approach.

## 7 Operating System Functions

In relation to virtual memory, describe the terms segment, page and translation lookaside buffer (TLB).

The operating system for a microprocessor supports a virtual memory model which implements both segmentation and paging. The only hardware assistance for the virtual memory system in the microprocessor is an on-chip TLB.

Outline the data structures held by the operating system.
[5 marks]
Describe the actions of the operating system in response to an address exception due to not matching the address issued by the processor in the TLB. [5 marks]

How can the operating system use access permissions to aid its page replacement policy?
[4 marks]

## 8 Data Structures and Algorithms

A directed graph of $n$ nodes numbered $1,2, \ldots, n$ can be represented by an $n \times n$ adjacency matrix $G_{1}$, where $G_{1}[i, j]$ is true if there is an edge connecting node $i$ to node $j$, and $G_{1}[i, j]$ is false otherwise.

By extension, define $G_{k}$ to be that matrix such that $G_{k}[i, j]$ is true if there is a path of length $\leqslant k$ connecting node $i$ to node $j$, and $G_{k}[i, j]$ is false otherwise.

Describe an algorithm to generate $G_{2}$ from $G_{1}$.
[12 marks]
How could this algorithm be used to generate the transitive closure of a graph given its adjacency matrix?

What is the cost of this transitive closure algorithm in terms of $n$ and $m$, where $m$ is the maximum path length in the transitive closure?

## 9 Graphics II

When scan-converting items for display, a Z-buffer is sometimes used to avoid some sorting. Outline its operation and limitations.

The use of an A-buffer will improve matters. Explain why.

## 10 Numerical Analysis I

What is meant by the term loss of significance? What is the essential difference between the terms condition and stability in numerical analysis? Define the term machine epsilon and explain why it is an important parameter.

Use the recurrence formula

$$
\cos [(k+1) \theta]=2 \cos \theta \cos [k \theta]-\cos [(k-1) \theta]
$$

with starting values $\cos 0=1, \cos \theta=\frac{1}{\sqrt{2}}+\varepsilon$ to evaluate $\cos 2 \theta$ and show that loss of significance occurs.

Evaluate $\cos 3 \theta$ and $\cos 4 \theta$, ignoring terms $O\left(\varepsilon^{3}\right)$. On this evidence, comment on the stability of the formula.

