COMPUTER SCIENCE TRIPOS Part II (General)
DIPLOMA IN COMPUTER SCIENCE

Monday 31 May 1993 1.30 to 4.30

Paper 10 (Paper 1 of Diploma in Computer Science)

Answer **five** *questions.*
Submit the answers in five **separate** *bundles each with its own cover sheet.*
Write on **one** *side of the paper only.*

1 Modula-2

Define the terms *opaque type* and *concrete type*. [5 marks]

The following is a shortened version of one of the definition modules described in the Modula-2 user manual:

```
DEFINITION MODULE Processes;
TYPE
  Semaphore;
Procedure InitSemaphore (VAR a:Semaphore);
Procedure Signal (s:Semaphore);
Procedure Wait (s:Semaphore);
.
.
END Processes.
```

Provide a suitable specification for type `Semaphore` as it might appear in the associated implementation module. [5 marks]

Write a fragment of Modula-2 code which uses a variable of type `Semaphore` and exploits the three procedures given above. Explain what the code does when each of the three procedures is invoked. [10 marks]

2 Modula-2

What is a Modula-2 *union*? [3 marks]

A Modula-2 program includes the following declarations:

```
TYPE
  SymbType  = (plus, minus, multiply, divide);
  NodeType  = (NumNode, OpNode);
  NumType   = RECORD val : CARDINAL END;
  PtrToNode = POINTER TO Node;
  OpType    = RECORD op : SymbType; f, s : PtrToNode END;
```

The objective is to be able to include assignment statements like:

```
test := MakeOpNode (multiply, MakeNumNode(4),
          MakeOpNode (minus, MakeNumNode(7), MakeNumNode(2)));
```

The variable `test` is of type `Node`, a record in which one field is either of type `NumType` or of type `OpType`, the latter representing a dyadic operator together with pointers to its two operands.

The procedure `MakeNumNode` takes a single `CARDINAL` parameter and returns a pointer to a `Node` which includes a `NumType` field. The procedure `MakeOpNode` returns a pointer to a `Node` which includes an `OpType` field.

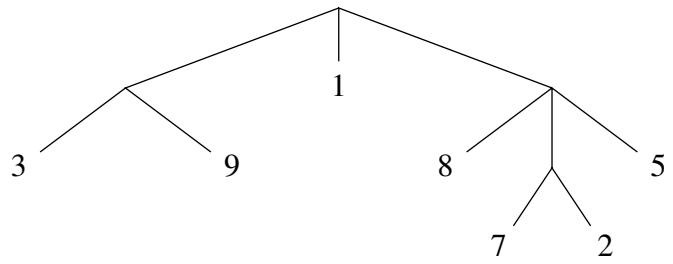
The effect of the example assignment statement is to assign to `test` a syntax tree which represents the expression `4*(7-2)`.

Provide a suitable declaration for type `Node`. [5 marks]

Write the procedures `MakeNumNode` and `MakeOpNode`. [6 marks each]

3 Common Lisp

Consider trees that have two kinds of nodes. A node is either a **leaf**, labelled by a number, or a **branch**, and has one or more subtrees. For example:



One imagines that the edges from each branch node are numbered from left to right starting from 0. A list of these numbers thus designates the path from the root to a node. In the tree shown above, the path (2 1 1) designates the path to the node labelled 2.

(*a*) Describe a good representation for such trees in Lisp. [3 marks]

(*b*) Write a Lisp function `getnode` such that (`getnode` *path tree*) returns the node of *tree* designated by *path*, assuming that the tree contains such a node. [5 marks]

(*c*) Write a Lisp function `maxpath` such that (`maxpath` *tree*) returns the maximum of the leaf nodes in the tree, together with the path to that node. For the tree shown above, `maxpath` should return 9 as the maximum and (0 1) as the path. [12 marks]

4 Programming Language Compilation

Discuss the merits of translating the abstract syntax tree representation of a program into assembly language by means of

(*a*) an *ad hoc* recursive tree walking program [10 marks]

(*b*) an algorithm based on tables automatically generated from tree translation rules [10 marks]

5 Operating Systems

Contrast UNIX pipes with a general, asynchronous message-passing facility as a basis for inter-process communication between processes which run in separate address spaces. [20 marks]

6 Operating System Functions

Describe the functionality you would expect to find in the file system directory service of a multi-user operating system. [10 marks]

Describe two ways in which multiple names for the same file can be supported, and what problems arise as a result. [10 marks]

7 Data Structures and Algorithms

Show that comparison-based sorting uses at best about $n \log n$ comparisons if there are n things to be sorted. [5 marks]

Compute the expected inefficiency ratio from using linear insertion as against an $O(n \log n)$ sort on lists of 16 and 32 objects. This is the ratio by which the expected number of comparisons exceeds the theoretical minimum. [5 marks]

Show that binary insertion may reasonably be expected to be an $O(n \log n)$ sort. [5 marks]

About how many comparisons would you expect to take place when sorting 1024 7-bit values by binary insertion? [5 marks]

8 Data Structures and Algorithms

Describe and justify an algorithm for determining the length of the shortest path between all pairs of vertices in a directed graph with lengths. [8 marks]

Design and provide an estimate of the size of a data structure by which the actual paths could be retrieved after the calculation above. [12 marks]

9 Graphics I

RasterOp is the name given to an operation which generates, from a number of rasters of pixel values, another raster of pixel values.

Describe suitable versions that could be used

(a) to move a window on screen while preserving background [12 marks]

(b) to blend two images in proportions given by a mask [8 marks]

10 Numerical Analysis I

In the IEEE binary standard (*IEEE* 754) what do the parameters p (precision), e_{min} and e_{max} specify? How is the value of an exponent e stored? [3 marks]

Explain the terms *normalised number*, *denormal number*, *hidden bit* and *NaN*. [4 marks]

In terms of the stored bit-pattern, how can each of the following be recognised: ± 0, $\pm\infty$, denormal number, NaN. [4 marks]

Suppose for some special-purpose hardware that a floating-point implementation is to be provided using only one byte for each representable number. Suppose also that, as far as possible, the principles of IEEE binary arithmetic are to be adhered to. If a sign bit of 0 represents a positive number, $p = 4$, $e_{min} = -2$ and $e_{max} = 3$ what should the following bit patterns represent?

00000000 00000001 00110000 11110000 11110001 [5 marks]

Consider the evaluation under IEEE arithmetic of the functions

$$(a) \quad \frac{x^2 + 2}{x^2 + 1} \qquad (b) \quad \ln(x^2 + 2)$$

where x and the function values are representable numbers, but x^2 is not. Show how you would formulate the evaluation of (a) and (b) to avoid this problem. [4 marks]

11 Discrete Mathematics

Let $g : A \to B$ be a function with domain A and range B. Show that the relation R defined by

$$xRy \Leftrightarrow g(x) = g(y)$$

is an equivalence relation on A. [4 marks]

Let $f(n, r)$ be the number of surjections from a set A having n elements to a set B having r elements. Show that

$$f(n, r) = r\big(f(n-1, r-1) + f(n-1, r)\big). \quad \text{[8 marks]}$$

Evaluate $f(n, r)$ in the cases:

(a) $r = 2$ [3 marks]

(b) $r = (n-1)$ [5 marks]

12 Proving Programs Correct

Explain the difference between partial and total correctness and give examples to illustrate the difference. [4 marks]

Does the usual FOR-rule for partial correctness work for total correctness? Justify your answer. [4 marks]

Does the usual WHILE-rule for partial correctness work for total correctness? Justify your answer. [4 marks]

Explain why it is necessary to assume that expressions are free of *side effects* for the Assignment Axiom to be sound. [4 marks]

Explain how arrays can be reasoned about using Hoare logic and discuss some of the pitfalls of naïve approaches. [4 marks]