COMPUTER SCIENCE TRIPOS Part II – 2020 – Paper 8

15 Types (nk480)

- (a) Recall that in constructive logic, logical negation is defined using implication and falsehood as $\neg A \triangleq A \supset \bot$.
 - (i) Does $A \supset \neg \neg A$? If so, give a simply-typed lambda-term corresponding to this implication.
 - (*ii*) Does $\neg \neg A \supset A$? If so, give a simply-typed lambda-term corresponding to this implication.
 - (*iii*) Does $\neg \neg \neg A \supset \neg A$? If so, give a simply-typed lambda-term corresponding to this implication.

[5 marks]

(b) (i) Give the typing rules for Peano natural numbers and their eliminator.

[2 marks]

- (ii) Using the rules given above, define the addition function. [3 marks]
- (iii) Let a binary tree be either a leaf Leaf or a node Node(l,x,r) where l and r are subtrees, and x is a natural number. Give typing rules for trees corresponding to this prose description, including an eliminator.

[3 marks]

- (iv) Using the rules given above, define a function size which takes a binary tree and returns the total number of nodes in the tree. [5 marks]
- (c) The zip function takes two lists, and returns a list of pairs of the elements as output. Suppose we see the following Agda type declaration for zip:

$$zip: \forall \{AB: Set\} \rightarrow \{n: Nat\} \rightarrow \mathsf{Vec}\,A\,n \rightarrow \mathsf{Vec}\,B\,n \rightarrow \mathsf{Vec}\,(A \times B)\,n$$

Explain what this means in terms of how to call the function, and what properties the result has. [2 marks]