8 Discrete Mathematics (MPF)

(*a*) Let $R \subseteq X \times Y$ and $P \subseteq Y$ for sets X and Y.

Prove that

$$\forall\, y \in Y.\ \Big(\ \big[\,(\exists\, x \in X.\ x\,R\,y) \Rightarrow y \in P\,\big] \iff \big[\,\forall\, x \in X.\ (x\,R\,y \Rightarrow y \in P)\,\big]\ \Big)$$

[6 marks]

(*b*) Define the notions of

(*i*) injective function between two sets [1 mark]

(*ii*) surjective function between two sets [1 mark]

(*c*) Let $\mathbb{N}_+ = \{n \in \mathbb{N} \mid n > 0\}$ and define the function $e : \mathbb{N} \times \mathbb{N} \to \mathbb{N}_+$ by

$$e(m, n) = 2^m(2n + 1)$$

Without using the Fundamental Theorem of Arithmetic, prove that e is

(*i*) injective [4 marks]

(*ii*) surjective [8 marks]

You may use any other standard results provided that you state them clearly.