7 Discrete Mathematics (MPF)

(*a*) Find all solutions in $\mathbb{Z}_{187}$ of the following congruence

$$x^2 + 5x + 6 \equiv 0 \pmod{187}$$

Justify your answer. [6 marks]

(*b*) For $\ell \in \mathbb{N}$, let $[\ell] = \{i \in \mathbb{N} \mid i < \ell\}$.

(*i*) Prove that, for all $\ell, m \in \mathbb{N}$, $[m] \times [\ell] \cong [m \cdot \ell]$ [3 marks]

(*ii*) Prove that, for all $\ell, m \in \mathbb{N}$, $[m] \uplus [\ell] \cong [m + \ell]$ [3 marks]

(*iii*) For $m, n \in \mathbb{N}$, define $\oplus$ by

$$[m] \oplus [0] = [m] \quad \text{and} \quad [m] \oplus [n+1] = \big([m] \oplus [n]\big) \uplus [1]$$

Prove that, for all $\ell, m \in \mathbb{N}$,

$$[m] \oplus [\ell] \cong [\ell] \oplus [m]$$ [8 marks]

You may use any standard results provided that you state them clearly.