Let the pure CCS processes $P_1, P_2, Q_1$ and $Q_2$ be as follows.

\[
P_1 \overset{\text{def}}{=} a.P_1 + \bar{b}.Q_1 \\
Q_1 \overset{\text{def}}{=} b.P_1 \\
P_2 \overset{\text{def}}{=} b.Q_2 \\
Q_2 \overset{\text{def}}{=} c.Q_2 + \bar{b}.P_2
\]

The transition system from $(P_1 \parallel P_2) \setminus \{b\}$ is as follows.

\[
\begin{array}{ccc}
\tau & a & \tau \\
(P_1 \parallel P_2) \setminus \{b\} & \cup & (Q_1 \parallel Q_2) \setminus \{b\}
\end{array}
\]

(a) Give full derivations for the two transitions that start from $(P_1 \parallel P_2) \setminus \{b\}$. [5 marks]

(b) The full modal-$\mu$ calculus has the syntax

\[
A ::= T \mid S \mid \neg A \mid A_1 \land A_2 \mid A_1 \lor A_2 \mid \langle a \rangle A \mid [a]A \mid \nu X.A \mid \mu X.A \mid X,
\]

where $S$ is an arbitrary set of states. Give a semantics to closed formulas \textit{without} using the abbreviations $\mu X.A \equiv \neg \nu X.\neg A[\neg X/X]$ and $[a]A \equiv \neg \langle a \rangle \neg A$. What condition must be placed on the occurrence of variables and why? [5 marks]

(c) Prove that the operation

\[
X \mapsto [a]X
\]

is $\cap$-continuous. [5 marks]

(d) Give a modal-$\mu$ formula that is satisfied by a process if, and only if, it is bisimilar to the process $(P_1 \parallel P_2) \setminus \{b\}$. You may assume that the process is only capable of actions labelled $a, c$ and $\tau$. [5 marks]