13 Hoare Logic and Model Checking (DPM)

Let \( AP \) be a set of atomic propositions, ranged over by \( p, q, \) and so on. Recall the grammar of Computation Tree Logic (CTL) path and state formulae:

\[
\phi, \psi, \xi ::= \Diamond \Phi \mid \Box \Phi \mid \bigcirc \Phi \mid \Phi \text{ UNTIL } \Psi \\
\Phi, \Psi, \Xi ::= \top \mid \bot \mid p \mid \Phi \land \Psi \mid \Phi \lor \Psi \mid \Phi \Rightarrow \Psi \mid \neg \Phi \mid \forall \phi \mid \exists \phi
\]

(a) Fix a CTL model \( M = \langle S, S_0, \rightarrow, L \rangle \). Suppose \( \phi \) is a CTL path formula, \( \Phi \) is a CTL state formula, \( s \) is a state in \( S \), and \( \pi \) is an infinite path of states of \( S \).

Define the two satisfaction relations \( M, \pi \models \phi \) and \( M, s \models \Phi \), explaining fully any notation that you use and any auxiliary definitions that you make.

[5 marks]

(b) Suppose \( p, q, \) and \( r \) are atomic propositions taken from the set \( AP \). Suppose also that we define a CTL model \( M = \langle S, S_0, \rightarrow, L \rangle \), where:

\[
S = \{s_0, s_1, s_2, s_3, s_4\} \quad S_0 = \{s_0, s_1\} \\
\rightarrow = \{(s_i, s_j) \mid i + j \text{ is even, for all } 0 \leq i \leq 4 \text{ and } 0 \leq j \leq 4\} \\
L(s_0) = L(s_2) = L(s_4) = \{p\} \quad L(s_3) = \{q\} \quad L(s_1) = \{q, r\}
\]

For each of the following, identify the set of all states \( s \in S \) for which it holds:

(i) \( M, s \models \forall \Box p \),

(ii) \( M, s \models \exists \Diamond q \),

(iii) \( M, s \models \exists \bigcirc (p \land r) \)

Explain fully how you computed your answer in each case. [6 marks]

(c) Define what it means for two CTL state formulae \( \Phi \) and \( \Psi \) to be semantically equivalent, written \( \Phi \equiv \Psi \). [3 marks]

(d) Show that \( (\Phi \lor \Psi) \land \Xi \) and \( (\Phi \land \Xi) \lor (\Psi \land \Xi) \) are semantically equivalent. [6 marks]