13 Hoare Logic and Model Checking (DPM)

Let AP be a set of atomic propositions, ranged over by p, q, and so on. Recall the grammar of Computation Tree Logic (CTL) path and state formulae:

$$\phi, \psi, \xi ::= \Diamond\Phi \mid \Box\Phi \mid \bigcirc\Phi \mid \Phi \texttt{ UNTIL } \Psi$$
$$\Phi, \Psi, \Xi ::= \top \mid \bot \mid p \mid \Phi \wedge \Psi \mid \Phi \vee \Psi \mid \Phi \Rightarrow \Psi \mid \neg\Phi \mid \forall\phi \mid \exists\phi$$

(*a*) Fix a CTL model $\mathcal{M} = \langle S, S_0, \rightarrow, L\rangle$. Suppose ϕ is a CTL path formula, Φ is a CTL state formula, s is a state in S, and π is an infinite path of states of S.

Define the two satisfaction relations $\mathcal{M}, \pi \models \phi$ and $\mathcal{M}, s \models \Phi$, explaining fully any notation that you use and any auxiliary definitions that you make.
[5 marks]

(*b*) Suppose p, q, and r are atomic propositions taken from the set AP. Suppose also that we define a CTL model $\mathcal{M} = \langle S, S_0, \rightarrow, \mathcal{L}\rangle$, where:

$$S = \{s_0, s_1, s_2, s_3, s_4\} \quad S_0 = \{s_0, s_1\}$$
$$\rightarrow\, = \{(s_i, s_j) \mid i + j \text{ is even, for all } 0 \leq i \leq 4 \text{ and } 0 \leq j \leq 4\}$$
$$\mathcal{L}(s_0) = \mathcal{L}(s_2) = \mathcal{L}(s_4) = \{p\} \quad \mathcal{L}(s_3) = \{q\} \quad \mathcal{L}(s_1) = \{q, r\}$$

For each of the following, identify the set of all states $s \in S$ for which it holds:

(*i*) $\mathcal{M}, s \models \forall\Box p$,

(*ii*) $\mathcal{M}, s \models \exists\Diamond q$,

(*iii*) $\mathcal{M}, s \models \exists \bigcirc (p \wedge r)$

Explain fully how you computed your answer in each case. [6 marks]

(*c*) Define what it means for two CTL state formulae Φ and Ψ to be semantically equivalent, written $\Phi \equiv \Psi$. [3 marks]

(*d*) Show that $(\Phi \vee \Psi) \wedge \Xi$ and $(\Phi \wedge \Xi) \vee (\Psi \wedge \Xi)$ are semantically equivalent.
[6 marks]