8 Mathematical Methods for Computer Science (RJG)

(a) (i) State the central limit theorem. [2 marks]

(ii) Consider a binomially distributed random variable $T$ with parameters $\text{Bin}(n,p)$ where $n$ is a positive integer and $0 < p < 1$. Using the central limit theorem derive an approximation to the probability $P(T > d)$ where $d \in (0, n)$ and where $n$ is sufficiently large. [4 marks]

(b) Let $(X_n)_{n \geq 1}$ be a Markov chain on the states $\{0, 1, 2\}$ with transition matrix

\[
P = \begin{pmatrix}
0 & 0 & 1 \\
0 & 1-\alpha & \alpha \\
1-\alpha & \alpha & 0
\end{pmatrix}
\]

where $0 < \alpha < 1$.

(i) Draw the state space diagram for the Markov chain $X_n$. [2 marks]

(ii) Explain why $X_n$ is an irreducible, recurrent and aperiodic Markov chain. [6 marks]

(iii) Define an equilibrium distribution $\pi = (\pi_0, \pi_1, \pi_2)$ for the Markov chain $X_n$ and determine $\pi$. [6 marks]