(a) The operating system typically provides each process with the illusion that it runs in a contiguous piece of memory. State the problem of external fragmentation in memory where processes have variably sized memory partitions. Describe how paged virtual memory solves this problem, and any time and space costs it introduces. [4 marks]

(b) Consider a 64-bit machine architecture providing 48-bit virtual addressing where the operating system uses a 4-level page table structure where a page is 4096 bytes and each page table entry is 8 bytes in size.

(i) Show how the virtual address \(0x00be.efc0.ffee\) is mapped to a physical address using the 4-level page table. You should give the size of each level in the page table in terms of both bytes and entries, as well as the size of a page table. [6 marks]

(ii) Assume a memory access takes 40 ns, and the machine provides a Translation Lookaside Buffer (TLB) with a hit rate of 90% and a search time of 10 ns. What is the effective memory access time? [4 marks]

(iii) Assume that a naive operating system designer instead proposes to use only a single level of page table structure. Show how this will affect both the space overheads of paging and the effective memory access time. Indicate whether a single level of page table is practical in this system. [3 marks]

(c) Consider a system where each process has three distinct memory areas requiring distinct access permissions. How do the space and time overheads due to paging change if the system moves from using small (e.g., \(2^{12}\) byte) pages to using large (e.g., \(2^{22}\) byte) pages? [3 marks]