10 Discrete Mathematics (IML)

For each of the following languages over the alphabet \( \{a, b\} \), state with justification whether the language is regular or not. \( m \) and \( n \) are natural numbers.

(a) \( L_1 \) is the set of all strings with the number of \( a \)'s in each being divisible by 3 and the number of \( b \)'s being divisible by 7. \([4 \text{ marks}]\)

(b) \( L_2 = \{a, b\} \) \([4 \text{ marks}]\)

(c) \( L_3 = \{a^m b^n \mid m \neq n\} \) \([4 \text{ marks}]\)

(d) \( L_4 = \{uvw^Rv \mid \text{nonempty strings } u, w, v \in \{a, b\}^*\} \)

\( w^R \) is the string obtained by reversing the string \( w \). \([4 \text{ marks}]\)

(e) \( L_5 = \{a^n \mid \text{where there are twin primes } p, p + 2, \text{ with } p > n\} \)

Twin primes are pairs of primes which differ by 2, such as 5 and 7, or 17 and 19. It has been conjectured – but never proven – that there are infinitely many twin primes. \([4 \text{ marks}]\)