10 Algorithms (DJW)

(a) Consider a directed acyclic graph with \( V \) vertices and \( E \) edges.

(i) What is meant by a total order on the vertices consistent with the edges? [2 marks]

(ii) Describe an \( O(E + V) \) algorithm to compute such a total order. [3 marks]

(b) Consider a directed graph with non-negative edge costs and with a given start vertex \( s \).

(i) Dijkstra’s algorithm computes distances from \( s \) to every other vertex. Give pseudocode for Dijkstra’s algorithm. [4 marks]

(ii) Dijkstra’s algorithm can be implemented using a Fibonacci heap. State the complexity of using this implementation. Justify your answer carefully. [Note: Your answer should include mention of amortized costs.] [4 marks]

(c) Consider a directed acyclic graph with non-negative edge costs and with a given start vertex \( s \).

(i) Devise an algorithm to compute distances from \( s \) in \( O(E + V) \) time. Justify why your algorithm is correct. [4 marks]

(ii) Explain, with an example, why Dijkstra’s algorithm might take \( \Omega(V \log V) \) time. [3 marks]