(a) Explain the scenario in which a strictness analyser is used to optimise a program. Your answer should consider the following: for what languages strictness optimisation is useful, where it is beneficial to be able to place strict or non-strict annotations on a program (seeing the strictness analyser as a black-box oracle), and how such annotations can safely allow an optimiser to represent strict or non-strict values differently at run time. Give an example program which has different run-time space complexity before and after strictness optimisation. [5 marks]

(b) One implementation for a strictness analyser determines strictness functions associated with each user-defined or built-in function. Given a user-defined function taking $n$ integer arguments to an integer result, state the domain and range of its associated strictness function. How can such a strictness function be used to produce the strict or non-strict annotations in Part (a)? [3 marks]

(c) Give a data structure suitable for representing strictness functions within a strictness analyser. Can ordinary functions be used? Would your data structure represent strictness functions $\lambda(x, y, z). x \land (y \lor z)$ and $\lambda(x, y, z). (x \land y) \lor (x \land z)$ differently? Would these two strictness functions enable different strictness optimisations in Part (a)? [4 marks]

(d) Give the strictness functions for the following source-language functions.

(i) The built-in addition and 3-argument conditional functions. [2 marks]

(ii) A built-in parallel-if function, which evaluates all its three arguments in parallel, and returns a result as soon as enough of its arguments terminate. This includes returning value $v$ when the second and third arguments evaluate to $v$ even if the first argument is still computing. [2 marks]

(iii) The user-defined function $f$ defined by

$$f(x, y, z, t, u) =$$
$$\text{if } x=0 \text{ then } y \text{ else } f(x-1, t+2, u+3, y*4, z*5);$$

[4 marks]