6 Digital Signal Processing (MGK)

(*a*) Let δ be the Dirac delta function and $T, b > 0$ be time intervals. Give the Fourier transform

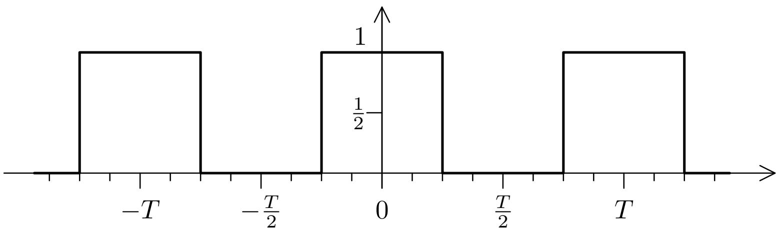
$$X(f) = \int_{-\infty}^{\infty} x(t)\, \mathrm{e}^{-2\pi \mathrm{j} f t}\, \mathrm{d}t$$

of the following two functions:

(*i*) $x(t) = c_T(t)$, where $c_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ [3 marks]

(*ii*) $x(t) = r_b(t)$, where $r_b(t) = \begin{cases} 1 & \text{if } |t| < b \\ \frac{1}{2} & \text{if } |t| = b \\ 0 & \text{otherwise} \end{cases}$ [5 marks]

(*b*) Consider this periodic, binary, square-wave clock signal $p(t)$, with period T, duty cycle 0.5 and maximum amplitude 1:

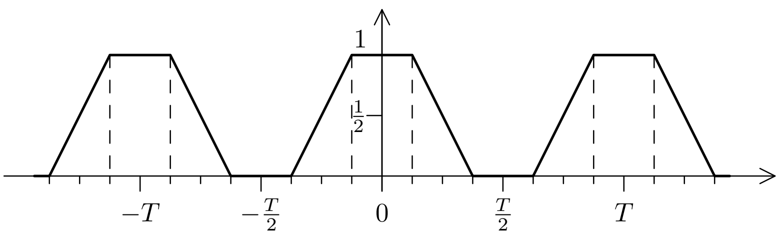


Show that its Fourier transform is

$$P(f) = \frac{1}{2}\delta(f) + \frac{1}{2\pi} \cdot \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{2k+1}{T}\right) \cdot \frac{(-1)^k}{k + \frac{1}{2}}.$$

Hint: Use the answers from part (*a*). [8 marks]

(*c*) Real-world digital signals need some time to transition between low and high. What is the Fourier transform of the periodic, trapezoid-wave clock signal $q(t)$, shown below, with period T and transition time $T/4$?



[4 marks]