6 Digital Signal Processing (MGK)

(a) Let \( \delta \) be the Dirac delta function and \( T, b > 0 \) be time intervals. Give the Fourier transform
\[
X(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi jft} \, dt
\]
of the following two functions:

(i) \( x(t) = c_T(t) \), where \( c_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \) \[3 \text{ marks}\]

(ii) \( x(t) = r_b(t) \), where \( r_b(t) = \begin{cases} 
1 & \text{if } |t| < b \\
\frac{1}{2} & \text{if } |t| = b \\
0 & \text{otherwise}
\end{cases} \) \[5 \text{ marks}\]

(b) Consider this periodic, binary, square-wave clock signal \( p(t) \), with period \( T \), duty cycle 0.5 and maximum amplitude 1:

Show that its Fourier transform is
\[
P(f) = \frac{1}{2} \delta(f) + \frac{1}{2\pi} \cdot \sum_{k=-\infty}^{\infty} \delta \left( f - \frac{2k + 1}{T} \right) \cdot \frac{(-1)^k}{k + \frac{1}{2}}.
\]

*Hint:* Use the answers from part (a). \[8 \text{ marks}\]

(c) Real-world digital signals need some time to transition between low and high. What is the Fourier transform of the periodic, trapezoid-wave clock signal \( q(t) \), shown below, with period \( T \) and transition time \( T/4 \)?

\[4 \text{ marks}\]