6 Digital Signal Processing (MGK)

(a) Let $\delta$ be the Dirac delta function and $T, b > 0$ be time intervals. Give the Fourier transform

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi jft} dt$$

of the following two functions:

(i) $x(t) = c_T(t)$, where $c_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ [3 marks]

(ii) $x(t) = r_b(t)$, where $r_b(t) = \begin{cases} 
1 & \text{if } |t| < b \\
\frac{1}{2} & \text{if } |t| = b \\
0 & \text{otherwise}
\end{cases}$ [5 marks]

(b) Consider this periodic, binary, square-wave clock signal $p(t)$, with period $T$, duty cycle 0.5 and maximum amplitude 1:

Show that its Fourier transform is

$$P(f) = \frac{1}{2} \delta(f) + \frac{1}{2\pi} \cdot \sum_{k=-\infty}^{\infty} \delta \left( f - \frac{2k + 1}{T} \right) \cdot \frac{(-1)^k}{k + \frac{1}{2}}.$$  

Hint: Use the answers from part (a). [8 marks]

(c) Real-world digital signals need some time to transition between low and high. What is the Fourier transform of the periodic, trapezoid-wave clock signal $q(t)$, shown below, with period $T$ and transition time $T/4$?

[4 marks]