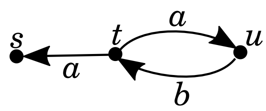
COMPUTER SCIENCE TRIPOS Part II – 2016 – Paper 9

14 Topics in Concurrency (JMH)

(*a*) For each of the following modal-μ assertions, write down the set of states in the following transition system which satisfy the given assertion.



(*i*) $[a]\langle b\rangle T$

(*ii*) $\langle a\rangle[b]F$

(*iii*) $\nu X.\langle -\rangle X$

(*iv*) $\mu X.\langle -\rangle X$

(*v*) $\nu X.\langle -\rangle\langle -\rangle X$ [5 marks]

(*b*) Let the set of states of an arbitrary transition system be S. The operation $\varphi : \mathcal{P}(S) \to \mathcal{P}(S)$ is defined, for a set of states $X \subseteq S$, as follows:

$$\varphi(X) = [-]X = \{y \ : \ \forall x \forall a.\ \text{if } y \xrightarrow{a} x \text{ then } x \in X\}.$$

Prove that $x \in \varphi^n(\emptyset)$ if, and only if, all sequences (including the empty sequence) of transitions starting from state x are of length less than n. [6 marks]

(*c*) For the operation defined in part (*b*) there are transition systems for which

$$\bigcup_{n\in\omega} \varphi^n(\emptyset) \quad \neq \quad \mu X.[-]X$$

(*i*) What can you immediately infer about the operation φ? [2 marks]

(*ii*) Explain whether such transition systems can be finite. [2 marks]

(*iii*) Give an example of a transition system for which

$$\bigcup_{n\in\omega} \varphi^n(\emptyset) \quad \neq \quad \mu X.[-]X$$

[3 marks]

(*iv*) State when, in general, a state satisfies $\mu X.[-]X$. [2 marks]