6  Digital Signal Processing (MGK)

(a) Figures (i)–(viii) show eight different input vectors \( x \in \mathbb{C}^8 \). For each, identify one of figures (A)–(H) that shows the DFT output \( X \in \mathbb{C}^8 \) with \( X_k = \sum_{n=0}^{7} x_n \cdot e^{-2\pi j kn/8} \).

Briefly explain each choice. Real components are shown as circles. For non-real vectors, the imaginary components are shown in addition as crosses. [8 marks]

(b) Are these statements true or false? Explain your answers. [3 marks each]

(i) The system \( y_n = x_n + y_{n-1} \) has an impulse response with \( z \)-transform \( \frac{1}{1+z} \).

(ii) A continuous signal can only be reconstructed after sampling if the sampling frequency is larger than twice the highest frequency in the signal.

(iii) Convolution of a signal with a triangular window function causes its power spectrum to be multiplied with a \( \text{sinc}^3 \) function.

(iv) To convert the \( z \)-transform \( H(z) \) of the impulse response of any LTI filter into the \( z \)-transform of its step response, divide \( H(z) \) by \( 1 - z^{-1} \).