11 Security II (MGK)

(a) Why does the formal security definition for collision-resistant hash functions require a key $s$ and a security parameter $n$, even though most commonly used standard secure hash functions lack such input parameters? [4 marks]

(b) If $h_s : \{0, 1\}^* \rightarrow \{0, 1\}^{\ell(n)}$ is a collision-resistant hash function, do the following constructions $H_s$ also provide collision-resistant hash functions? Explain your answers. [2 marks each]

(i) $H_s(x) = h_s(x) \parallel x$ (i.e. append $x$)
(ii) $H_s(x) = h_s(x) \parallel \text{LSB}(x)$ (i.e. append least significant bit of $x$)
(iii) $H_s(x) = h_s(x | 1)$ (bitwise-or, i.e. set least significant bit of $x$ to 1)

(c) Use Euler’s theorem to calculate $5^{-1} \mod 8$. [4 marks]

(d) The standard Digital Signature Algorithm (DSA) uses a cyclic subgroup $G \subset \mathbb{Z}_p^*$ of the integers modulo a prime $p$, with prime order $q$, where $q$ divides $p - 1$.

(i) Give two advantages of using a multiplicative subgroup of prime order, as opposed to just using $\mathbb{Z}_p^*$, in cryptographic schemes based on the Discrete Logarithm problem. [2 marks]

(ii) Why is it possible to choose $q$ substantially smaller than $p$, and what is an advantage of doing so? [4 marks]