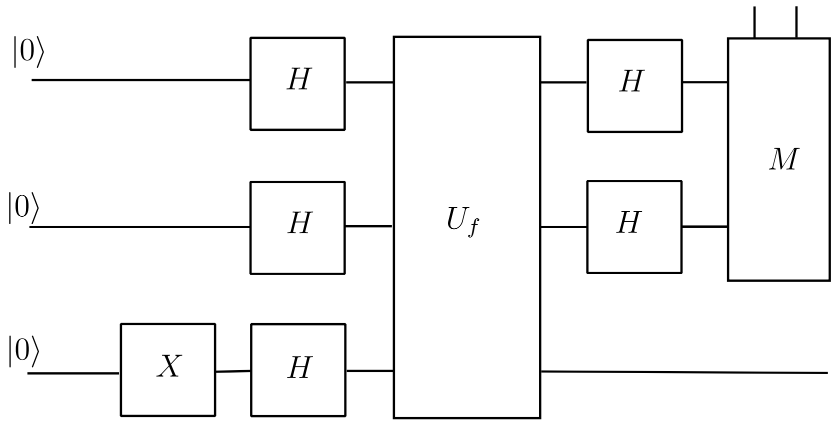
10 Quantum Computing (AD)

Let $f : \{0,1\}^2 \to \{0,1\}$ be a Boolean function of two inputs. Let U_f be the implementation of f as a unitary operator on 3 qubits defined by:

$$U_f|x\rangle|y\rangle|z\rangle = |x\rangle|y\rangle|z \oplus f(x,y)\rangle,$$

where $\oplus$ denotes the exclusive-or operation, and $|x\rangle|y\rangle|z\rangle$ is any computational basis state.

Consider the following circuit (a two-qubit version of the Deutsch-Josza circuit) in which X denotes a NOT gate, H denotes a Hadamard gate and M is a two-qubit measurement in the computational basis.



(a) Show that if f is a constant function, the outcome of the measurement M is 00 with probability 1. [6 marks]

(b) Show that if f is the XOR function, the outcome of the measurement M is 11 with probability 1. [6 marks]

(c) What are the probabilities of M measuring 00 and 11 respectively, if f is the Boolean AND function? [8 marks]