This question considers a language \( L \) which has integer variables \( V \), arithmetic expressions \( E \) and boolean expressions \( B \), along with commands \( C \) of the forms \( V := E \) (assignment), \( C; C' \) (sequencing), \( \text{IF} \ B \ \text{THEN} \ C \ \text{ELSE} \ C' \) (conditional) and \( \text{WHILE} \ B \ \text{DO} \ C \) (iteration).

\((a)\) Explain the syntax of the Hoare-logic partial-correctness formula \( \{P\} \ C \ \{Q\} \) and give a careful definition in English of when it is valid, that is, when \( \models \{P\} \ C \ \{Q\} \). [2 marks]

\((b)\) How does the definition of validity for the total-correctness formula \( [P] \ C \ [Q] \) differ? [1 mark]

\((c)\) Preconditions and postconditions in \( \{P\} \ C \ \{Q\} \) often make use of logical or auxiliary variables \( v \) in addition to program variables \( V \). Explain why this is useful illustrating your answer with a command \( C \) which satisfies \( \{T\} \ C \ \{R := X + Y\} \) but not \( \{X = x \land Y = y\} \ C \ \{R := x + y\} \). [3 marks]

\((d)\) Give the axioms and rules of an inference system \( \vdash \{P\} \ C \ \{Q\} \) for Hoare logic. [4 marks]

\((e)\) Are your rules sound? To what extent are they complete? [2 marks]

\((f)\) Give a formal proof, using your inference system, of
\[ \{X = x \land Y = 3\} \ X := X + 1 \ \{X - 1 = x \land Y < 10\} \]. [2 marks]

\((g)\) Consider the command \( C \) given by \( \text{WHILE} \ X > 0 \ \text{DO} \ (X := X - 1; \ Y := Y + 3) \), and let \( P \) be the precondition \( X = x \land Y = y \land x \geq 0 \). Give the strongest postcondition \( Q \) that you can establish. Give any invariant necessary to prove \( \{P\} \ C \ \{Q\} \) for your \( Q \). Explain briefly how the structure of the proof relates to the structure of \( C \). [6 marks]