2 Advanced Graphics (PB)

A force function $F : \mathbb{R}^3 \to \mathbb{R}$ takes a 3D point and returns a scalar representing a value of force. Force functions are the fundamental building blocks of metaball modelling.

We will build an implicit surface renderer which takes as input a set of force functions \{\(F_1(P), \ldots, F_n(P)\}\} and renders the set of all points \(P\) in space where the forces of the functions sum to a threshold: the 3D isosurface such that \(\sum F_i(P) = 0.5\).

(a) Using pseudocode, give a force function \(\text{Sphere}(P)\) which will render a unit sphere centred on \((0, 0, 0)\). [Figure 1] [2 marks]

(b) Using pseudocode, give a force function \(\text{Cube}(P)\) which will render an axis-aligned cube of edge length 2 centred on \((1, 1, -1)\). [Figure 2] [4 marks]

(c) You now pass both \(\text{Sphere}(P)\) and \(\text{Cube}(P)\) to your implicit surface renderer. Depending on your choice of force functions, the seam between the cube and the sphere may be a sharp edge (to within the tolerance of your polygonalization) or a smooth blend which merges gradually from one form into the other. Which will it be, and (briefly) why? [Figures 3 and 4] [2 marks]

(d) Provide alternate formulations of \(\text{Sphere}(P)\) and/or \(\text{Cube}(P)\) such that if you answered ‘smooth’ to Part (c) then your answer would now be ‘sharp’, or vice-versa. [4 marks]

A spatial distortion function \(S : \mathbb{R}^3 \to \mathbb{R}^3\) transforms one 3D point to another. If the points passed into the force function are modified by a spatial distortion function—that is, if we render \(F(S(P))\)—then the rendered isosurface will have a different shape.

For example, if we define \(S(P)\) as

```java
function Point S(P) {
    return new Point(P.x * 2, P.y / 2, P.z * 2);
}
```

then rendering the implicit surface of \(\text{Sphere}(S(P))\) will yield a tall, narrow ellipsoid along the \(Y\) axis. [Figure 5]

(e) Give a spatial distortion function \(S(P)\) such that rendering the isosurface of \(\text{Cube}(S(P))\) would render the cube centred at the origin and rotated 45 degrees around the \(X\) axis. [Figure 6]

\(Hint:\) a standard rotation matrix is \(\begin{pmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{pmatrix}\). [3 marks]
(f) Define $S(P)$ as

$$
\text{function Point } S(P) \{
\text{  return new Point(}
\text{      P.x / 4,}
\text{      P.y * 2 / sin(P.x * PI),}
\text{      P.z * 2);}
\text{  }
\}
$$

Describe and draw a sketch of the isosurface defined by $\text{Sphere}(S(P))$.

[5 marks]

Figures:

Figure 1: A sphere centred at $(0,0,0)$

Figure 2: A cube of edge length 2 centred at $(1,1,-1)$

Figure 3: A sharp join between sphere and cube

Figure 4: A smooth blending between sphere and cube

Figure 5: A vertical ellipsoid

Figure 6: A tilted cube centred at $(0,0,0)$