2 Advanced Graphics (PB)

A force function \( F : \mathbb{R}^3 \to \mathbb{R} \) takes a 3D point and returns a scalar representing a value of force. Force functions are the fundamental building blocks of metaball modelling.

We will build an implicit surface renderer which takes as input a set of force functions \( \{F_1(P), \ldots, F_n(P)\} \) and renders the set of all points \( P \) in space where the forces of the functions sum to a threshold: the 3D isosurface such that \( \sum F_i(P) = 0.5 \).

(a) Using pseudocode, give a force function \( \text{Sphere}(P) \) which will render a unit sphere centred on \((0,0,0)\). [Figure 1] [2 marks]

(b) Using pseudocode, give a force function \( \text{Cube}(P) \) which will render an axis-aligned cube of edge length 2 centred on \((1,1,-1)\). [Figure 2] [4 marks]

(c) You now pass both \( \text{Sphere}(P) \) and \( \text{Cube}(P) \) to your implicit surface renderer. Depending on your choice of force functions, the seam between the cube and the sphere may be a sharp edge (to within the tolerance of your polygonalization) or a smooth blend which merges gradually from one form into the other. Which will it be, and (briefly) why? [Figures 3 and 4] [2 marks]

(d) Provide alternate formulations of \( \text{Sphere}(P) \) and/or \( \text{Cube}(P) \) such that if you answered ‘smooth’ to Part (c) then your answer would now be ‘sharp’, or vice-versa. [4 marks]

A spatial distortion function \( S : \mathbb{R}^3 \to \mathbb{R}^3 \) transforms one 3D point to another. If the points passed into the force function are modified by a spatial distortion function—that is, if we render \( F(S(P)) \)—then the rendered isosurface will have a different shape.

For example, if we define \( S(P) \) as

\[
\text{function Point S(P) \{} \\
\quad \text{return new Point(P.x * 2, P.y / 2, P.z * 2);} \\
\}\n\]

then rendering the implicit surface of \( \text{Sphere}(S(P)) \) will yield a tall, narrow ellipsoid along the \( Y \) axis. [Figure 5]

(e) Give a spatial distortion function \( S(P) \) such that rendering the isosurface of \( \text{Cube}(S(P)) \) would render the cube centred at the origin and rotated 45 degrees around the \( X \) axis. [Figure 6]

\[
\text{Hint: a standard rotation matrix is } \begin{pmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{pmatrix}. \] [3 marks]
(f) Define $S(P)$ as

```javascript
function Point S(P) {
  return new Point(
    P.x / 4,
    P.y * 2 / sin(P.x * PI),
    P.z * 2);
}
```

Describe and draw a sketch of the isosurface defined by $Sphere(S(P))$.

[5 marks]

Figures:

- Figure 1: A sphere centred at $(0, 0, 0)$
- Figure 2: A cube of edge length 2 centred at $(1, 1, -1)$
- Figure 3: A sharp join between sphere and cube
- Figure 4: A smooth blending between sphere and cube
- Figure 5: A vertical ellipsoid
- Figure 6: A tilted cube centred at $(0, 0, 0)$