10 Information Theory (JGD)

(a) Consider a discrete memoryless channel whose input symbol source is a random variable \( X \in \{x_1, \ldots, x_J\} \) having probability distribution \( p(x_j) \), and whose output symbol (possibly corrupted) is a random variable \( Y \in \{y_1, \ldots, y_K\} \) (see figure below).

\[
\begin{array}{cc}
\text{Symbols} & \xrightarrow{\text{Source encoder}} \text{Decoder} & \text{Symbols} \\
X & \xrightarrow{\text{Channel}} & Y
\end{array}
\]

(i) Provide its channel matrix. [3 marks]

(ii) Give the average probability of correct reception, meaning the probability that the same symbol is emitted as was injected into the channel, averaged over all the cases. [3 marks]

(b) Show that convolution of any continuous signal with a Dirac delta function reproduces the signal. [4 marks]

(c) A frequency-shifting modulation of signals into different channels of a shared medium multiplies the baseband signal \( f(t) \) by a complex exponential carrier wave \( e^{ict} \) of some (channel-specific) frequency \( c \) to produce a passband \( f(t)e^{ict} \) (see figure below). Upon reception of such a passband, what process of demodulation would recover the original baseband?

\[
\begin{array}{cc}
\text{baseband} & \text{carrier signal} \\
\text{AM passband}
\end{array}
\]

[5 marks]

(d) Explain the “information diagram” of Gabor, and why the Uncertainty Principle gives it a quantal structure with an irreducible representation of the data. [5 marks]