1 Advanced Algorithms (TMS)

(a) State the zero-one principle in the context of sorting networks. \[2\text{ marks}\]

(b) For each of the following six comparison networks, state whether it is a sorting network or not. In each case, justify your answer. For the justification you may refer to standard results without giving a proof. \[9\text{ marks}\]

(c) Let \(n\) be an exact power of 2. Show how to construct an \(n\)-input, \(n\)-output comparison network of depth \(\log n\) in which the top output wire always carries the minimum input value and the bottom output wire always carries the maximum input value. \[4\text{ marks}\]

(d) (i) Prove that the number of comparators in any sorting network is \(\Omega(n \log n)\). \[4\text{ marks}\]

(ii) What does Part (d)(i) imply in terms of the depth of any sorting network? \[1\text{ mark}\]