5 Databases (TGG)

(a) Define the concept of a functional dependency. [3 marks]

(b) Suppose that relation $R$ has $m$ attributes. Give an upper bound on the number of functional dependencies that $R$ could satisfy (including trivial dependencies). [3 marks]

(c) Let $R(A, B, C, D, E)$ be a relational scheme with the following dependencies.

\[
\begin{align*}
A &\rightarrow C \\
B, C &\rightarrow D \\
A &\rightarrow E \\
B, D &\rightarrow C \\
C &\rightarrow E \\
E &\rightarrow D \\
E &\rightarrow B
\end{align*}
\]

Which, if any, of these dependencies are redundant? [4 marks]

(d) Suppose $R(A, B, C)$ is a relational schema with functional dependency $A \rightarrow B$. What can you deduce about the results of $\pi_{A, B}(R) \bowtie_A \pi_{A, C}(R)$? Justify your answer. [3 marks]

(e) Suppose $R(A, B, C)$ is a relational schema. In addition, you know that the following is always true in any correct database instance.

\[R = \pi_{A, B}(R) \bowtie_A \pi_{A, C}(R).\]

What can you deduce about the dependencies between attributes $A$, $B$, and $C$? Prove any of your claims. [7 marks]