

COMPUTER SCIENCE TRIPOS Part IA – 2016 – Paper 2

8 Discrete Mathematics (MPF)

(a) (i) Calculate two integers x and y satisfying $177x + 78y = 3$. [3 marks]

(ii) Describe all the integer pairs (x, y) that satisfy the above equation. [3 marks]

(b) Let $\mathbb{N}_{\geq 5} = \{n \in \mathbb{N} \mid n \geq 5\}$. Prove that: $\forall n \in \mathbb{N}_{\geq 5}. 2^n > n^2$. [6 marks]

(c) Let $C(X, Y) = \{S \subseteq X \mid S \cong Y\}$.

(i) For finite sets X and Y , what is the cardinality of $C(X, Y)$ in terms of that of X and Y ? [2 marks]

(ii) For elements a and b , and sets A and B such that $\{a, b\} \cap (A \cup B) = \emptyset$, consider the functions

• $f : C(A \cup \{a\}, B \cup \{b\}) \longrightarrow C(A, B) \uplus C(A, B \cup \{b\})$ given by

$$f(S) = \begin{cases} (0, S \setminus \{a\}) & , a \in S \\ (1, S) & , a \notin S \end{cases}$$

and

• $g : C(A, B) \uplus C(A, B \cup \{b\}) \longrightarrow C(A \cup \{a\}, B \cup \{b\})$ given by

$$g(i, S) = \begin{cases} S \cup \{a\} & , i = 0 \\ S & , i = 1 \end{cases}$$

Prove either that $g \circ f = \text{id}$ or that $f \circ g = \text{id}$. [6 marks]