1 Foundations of Computer Science (LCP)

(a) Write brief notes on functions as values and results in ML, illustrated with the help of the functionals `map` and `exists`. What functions can we obtain from these via currying? [6 marks]

(b) Consider the function `zarg` defined below:

\[
\text{fun } zarg f \ (\text{[]}, \ e) = e \\
\mid zarg f \ (x::xs, \ e) = f(x, \ zarg f \ (xs,e))
\]

Show that with the help of this function, it is possible to write an expression for the sum of a given list of integers. Then describe what `zarg` does in general. [4 marks]

(c) A polymorphic type of branching trees can be declared as follows. Note that the children of a branch node are given as a list of trees, and that only the leaf nodes carry labels.

\[
\text{datatype } 'a \ vtree = \text{Lf of } 'a \\
\mid \text{Br of } ('a \ vtree) \ list;
\]

(i) Write a function `flat t` that converts a given tree `t` of this type to a list of the labels (without eliminating duplicates). Your function should run in linear time in the size of the tree. [4 marks]

(ii) Write a function `count x t` that counts the number of times that `x` occurs as a label in `t`, but without first converting `t` to a list.

*Note:* Minimal credit will be given for solutions that use `flat`. [5 marks]

(iii) What is the type of `count`? [1 mark]

All ML code must be explained clearly and should be free of needless complexity.