2 Computer Systems Modelling (RJG)

(*a*) A Poisson process of rate λ has inter-event times $X_1, X_2, \dots$ that are independent Exponential random variables with parameter λ.

(*i*) Show that the random variables X_i for $i = 1, 2, \dots$ satisfy the *memoryless property*

$$\mathbb{P}(X_i > t + s | X_i > t) = \mathbb{P}(X_i > s)$$

where s and t are any positive real numbers. [2 marks]

(*ii*) Suppose that $T_1, T_2, \dots, T_n$ is an observed sequence of n consecutive event times. Describe what tests you could conduct of the hypothesis that the observed events arise from a Poisson process of some given rate λ. [6 marks]

(*b*) (*i*) Describe the M/G/1 queue model explaining the mathematical assumptions made and the required features of the queue. [4 marks]

(*ii*) Suppose that the arrival rate for the M/G/1 queue is λ customers per second, that the service rate is μ customers per second and assume that $\rho = \lambda/\mu < 1$. Define the utilization, U, of the server and show that $U = \rho$. [2 marks]

(*iii*) A *busy period* of the M/G/1 queue is a period of time which starts when the server becomes occupied and continues until the server is no longer occupied. An *idle period* is the period of time between consecutive busy periods. Explain how to find the mean length of an idle period. Using your result for the utilization of the server in part (*b*)(*ii*), derive an expression for the mean length, τ, of a busy period. [6 marks]