COMPUTER SCIENCE TRIPOS Part IA − 2015 − Paper 2

8 Discrete Mathematics (MPF)

(a) Prove that, for all natural numbers n,

$$n^{13} \equiv n \pmod{1365}$$

You may use any standard results provided that you state them clearly.

[5 marks]

(b) For n ranging over the natural numbers $\mathbb{N}$, let

Even(n) be the predicate $\exists\, k \in \mathbb{N}.\ n = 2 \cdot k$

and let

Odd(n) be the predicate $\exists\, l \in \mathbb{N}.\ n = 2 \cdot l + 1$

Prove that
$$\forall\, n \in \mathbb{N}.\ \text{Even}(n) \vee \text{Odd}(n)$$
by the Principle of Induction.

[5 marks]

(c) Let $F : A \longrightarrow\!\!\!\!\!\rightarrow B$ be a relation, from a set A to a set B.

(i) Define what it means for F to be a (total) function. [2 marks]

(ii) Prove that F is a function if, and only if, there exists a relation $G : B \longrightarrow\!\!\!\!\!\rightarrow A$ such that $\mathrm{id}_A \subseteq G \circ F$ and $F \circ G \subseteq \mathrm{id}_B$. [8 marks]