

9 Algorithms (TMS)

- (a) Consider the two standard representations of directed graphs: the adjacency-list representation and the adjacency-matrix representation. Find a problem that can be solved more efficiently in the adjacency-list representation than in the adjacency-matrix representation, and another problem that can be solved more efficiently in the adjacency-matrix representation than in the adjacency-list representation. [4 marks]
- (b) Prove or disprove (by giving a counter-example) the following claim: If a directed graph  $G$  contains a path from a vertex  $u$  to a vertex  $v$ , then any depth-first search must result in  $v.d \leq u.f$ , where  $.d$  is the discovery time and  $.f$  the finishing time. [4 marks]
- (c) We are given an undirected, connected graph  $G = (V, E)$  with edge-weights  $w : E \rightarrow \mathbb{R}^+$  and a minimum spanning tree  $T$  of  $G$ . How would you update your minimum spanning tree  $T$  in each of the following three cases? Specify the runtime of your algorithm and give a proof that the returned tree is indeed a minimum spanning tree.
- (i) We increase the weight of an edge  $e$  which is not in  $T$ . [3 marks]
- (ii) We decrease the weight of an edge  $e$  which is in  $T$ . [3 marks]
- (iii) We add a new edge  $e$  with weight  $w(e)$  to  $G$ . The weight  $w(e)$  is arbitrary, but for simplicity you may assume that after adding the edge  $e$  no two edges in  $G$  have the same weight. [6 marks]