10 Algorithms (TMS)

(a) State the Max-Flow Min-Cut Theorem. [2 marks]

(b) For an arbitrary integer \( k \geq 1 \), give an example of a flow network with at most five vertices on which the basic Ford-Fulkerson method takes at least \( k \) steps to terminate. [4 marks]

(c) Consider the following flow network \( G \):

![Flow Network Diagram]

Given an initial flow \( f \) with \( f(s,u) = f(u,w) = f(w,t) = 2 \), perform one iteration of Ford-Fulkerson; that is, draw the residual graph \( G_f \), specify an augmenting path in \( G_f \), and update the flow \( f \). Is this new flow a maximum flow? Justify your answer. [5 marks]

(d) Given an undirected, connected graph \( G = (V,E) \), the edge-connectivity of \( G \) is the size of a smallest set of edges \( X \subseteq E \) so that the graph \( G' = (V,E \setminus X) \) becomes disconnected.

(i) Describe an algorithm that computes the edge-connectivity of \( G \), and analyse its runtime and correctness. [7 marks]

(ii) Extend your algorithm so that it also returns a set \( X \) satisfying the conditions above. [2 marks]