5 Digital Signal Processing (MGK)

A discrete sequence $\{x_n\}$ can be converted into a continuous representation

$$\hat{x}(t) = t_s \cdot \sum_{n=-\infty}^{\infty} \delta(t - n \cdot t_s) \cdot x_n,$$

where $t_s$ is the sampling period.

(a) State two characteristic properties of Dirac’s $\delta$ function. [2 marks]

(b) Describe briefly how this representation helps to explain aliasing. [4 marks]

(c) Define three functions $h(t)$, such that convolving $\hat{x}(t)$ with $h(t)$ results in

(i) the output of an idealized analog-to-digital converter that holds the output voltage of each sample $x_n$ for the time interval from $t = n \cdot t_s$ until the next sample $x_{n+1}$ arrives at time $t = (n + 1) \cdot t_s$; [4 marks]

(ii) linear interpolation of $\{x_n\}$; [4 marks]

(iii) reconstruction of a signal $x(t)$ that was sampled as $x_n = x(n \cdot t_s)$, assuming that the Fourier transform of $x(t)$ is zero at any frequency $f$ with $|f|^{-1} \leq t_s$ or $|f|^{-1} \geq 2t_s$. [6 marks]