2 Computer Systems Modelling (RJG)

(a) Consider a Poisson process with rate $\lambda > 0$. Let $X_1$ be the time of the first event and let $X_i$ be the time between events $(i - 1)$-st and $i$ for $i = 2, 3, \ldots$

(i) Derive the joint probability distribution of $(X_1, X_2)$. [5 marks]

(ii) Let $S_n = \sum_{i=1}^{n} X_i$. Derive the probability density function of $S_n$ and give expressions for the mean and variance of $S_n$. [5 marks]

(b) (i) Describe what is meant by a FCFS $M/G/1$ queueing system. Your description should include a clear statement of the probabilistic assumptions. [5 marks]

(ii) Suppose that you are given a log of timestamps for the arrival and departure events observed in an alleged simulation of a FCFS $M/G/1$ queue with a given general service time distribution. Describe the statistical tests that you would perform on the logged data to test whether the modelling assumptions are satisfied. [5 marks]