8 Temporal Logic and Model Checking (MJCG)

From Wikipedia: "Tic-tac-toe (or Noughts and Crosses, `X`s and `O`s) is a paper-and-pencil game for two players, `X` and `O`, who take turns marking the spaces in a 3×3 grid. The player who succeeds in placing three respective marks in a horizontal, vertical, or diagonal row wins the game." For example, `X` is the first player in both example games shown below; the first game is won by the `X`, the second is drawn.

```
 | |X   O| |X   O| |X   O| |X   O| |X   O| |X   O| |X
-|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-
 | |     | |     | |     |O|     |O|     |O|O    |O|O
-|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-
 | |     | |    X| |    X| |    X| |X   X| |X   X|X|X

 | |     | |O    | |O   O| |O   O|X|O   O|X|O   O|X|O   O|X|O   O|X|O
-|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-
 |X|     |X|     |X|     |X|     |X|     |X|    X|X|    X|X|O   X|X|O
-|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-   -|-|-
 | |     | |     | |X    | |X    | |X    |O|X    |O|X    |O|X   X|O|X
```

(*a*) This part of the question asks you to define a Kripke structure $M = (S, S_0, R, L)$ to model Tic-tac-toe. Assume the set AP consists of atomic propositions $\texttt{Start}(p)$ and $\texttt{Has}(i, v)$. $\texttt{Start}(p)$ means player p starts, where $p \in \{0, 1\}$ represents a player: 0, 1 represent `O`, `X`, respectively. $\texttt{Has}(i, v)$ means space i contains value v, where $i \in \{1, \ldots, 9\}$ names a grid space and $v \in \{0, 1, 2\}$ represents the state of a space: 0, 1, 2 represent `O`, `X`, empty-space, respectively.

(*i*) Specify a suitable representation S of states. [2 marks]

(*ii*) Specify the set of initial states S_0. [2 marks]

(*iii*) Specify a transition relation R to model the moves in the game. [6 marks]

(*iv*) Specify a labelling function L to define which atomic propositions hold in each state. [2 marks]

(*b*) In a suitable temporal logic, which you should name, devise and explain a formula ψ such that $M \models \psi$ if and only if the first player can always win or draw, no matter how the second player plays. [8 marks]