COMPUTER SCIENCE TRIPOS Part II – 2014 – Paper 8

6 Digital Signal Processing (MGK)

(*a*) Consider a causal, order-2 digital filter with real-valued infinite impulse response sequence $h_0, h_1, h_2, \ldots$

(*i*) What is the z-transform $H(z)$ of this filter's impulse response? [2 marks]

(*ii*) Express $H(z)$ in terms of the locations c_1, c_2 of its two zeros and the locations d_1, d_2 of its two poles in $\mathbb{C}$. [4 marks]

(*iii*) Give a necessary condition for c_1, c_2, d_1, d_2 to ensure that $\{h_n\}$ has only real values. [4 marks]

(*iv*) If we operate that filter at sampling frequency f_s, what will its amplitude gain at frequency f be? [2 marks]

(*b*) A *notch filter* aims to suppress a single frequency f_c. One way of designing an order-2 notch filter, as in part (*a*), involves placing the zeros directly onto the unit circle, and the poles right next to them inside the unit circle, at distance $0 < \alpha < 1$ from 0:

$$c_1 = \mathrm{e}^{\mathrm{j}\omega}, \quad d_1 = \alpha \cdot c_1, \quad c_2 = \mathrm{e}^{-\mathrm{j}\omega}, \quad d_2 = \alpha \cdot c_2, \quad \text{with} \quad \omega = 2\pi f_\mathrm{c}/f_\mathrm{s}$$

(*i*) What is the z-transform of the impulse response of the resulting filter, written as a fraction of two polynomials of z^{-1}? [4 marks]

(*ii*) The *OxyMax* is a medical device designed in the United States. It processes a heart-beat signal with a sampling rate of $f_\mathrm{s} = 600$ Hz. It contains the following C function, which implements a notch filter, as in part (*b*)(*i*), to suppress in the input signal interference from the North American power grid at $f_\mathrm{c} = 60$ Hz:

```
double mains_notch(double sample) {
  static double x[4], y[4];
  static int n = 0;
  x[n&3] = sample;
  y[n&3] = sample + x[(n-1)&3] * b1 + x[(n-2)&3]
                  - y[(n-1)&3] * a1 - y[(n-2)&3] * a2;
  return y[n++&3];
}
```

The U.S. version initializes the constants used with `b1` $= -2\cos(\pi/5)$, `a1` $=$ `b1` $\times\, 0.9$ and `a2` $= 0.81$. What changed constant(s) will instead suppress the power-grid frequency at $f_\mathrm{c} = 50$ Hz for the European version? [4 marks]