

6 Logic and Proof (LCP)

- (a) Describe briefly the concept of a decision procedure, listing at least three separate examples of decidable theories. [4 marks]
- (b) Outline the basic ideas behind Fourier-Motzkin variable elimination, demonstrating them with reference to the following small set of constraints:

$$x + 2y \geq 10 \quad x + z \leq 5 \quad y \leq 3 \quad z - 2 \geq 0$$

[6 marks]

- (c) Call a clause *positive* if it consists of positive literals only. *Negative selection* is a refinement of resolution where two clauses can be resolved only if one of them is positive; if a clause contains any negative literals, then only one of those may be resolved with a literal in another (necessarily positive) clause. Negative selection reduces the number of combinations of literals to be compared, thereby improving performance. Consider the following set of clauses:

$$\{R(0), R(1)\} \quad \{P(h(z)), \neg R(z)\} \quad \{\neg P(x), \neg R(y)\}.$$

With negative selection, the first resolution step must involve $\{R(0), R(1)\}$, as no other positive clauses are available at the start.

- (i) If a set of clauses includes no positive clauses, can it be unsatisfiable? Justify your answer. [3 marks]
- (ii) Use resolution with negative selection to derive a contradiction from the clauses above. [7 marks]