6 Logic and Proof (LCP)

(a) Describe briefly the concept of a decision procedure, listing at least three separate examples of decidable theories. [4 marks]

(b) Outline the basic ideas behind Fourier-Motzkin variable elimination, demonstrating them with reference to the following small set of constraints:

\[
\begin{align*}
    x + 2y & \geq 10 \\
    x + z & \leq 5 \\
    y & \leq 3 \\
    z - 2 & \geq 0
\end{align*}
\]

[6 marks]

(c) Call a clause positive if it consists of positive literals only. Negative selection is a refinement of resolution where two clauses can be resolved only if one of them is positive; if a clause contains any negative literals, then only one of those may be resolved with a literal in another (necessarily positive) clause. Negative selection reduces the number of combinations of literals to be compared, thereby improving performance. Consider the following set of clauses:

\[
\begin{align*}
    \{R(0), R(1)\} & \quad \{P(h(z)), \neg R(z)\} & \quad \{\neg P(x), \neg R(y)\}.
\end{align*}
\]

With negative selection, the first resolution step must involve \(\{R(0), R(1)\}\), as no other positive clauses are available at the start.

\[\text{(i)}\] If a set of clauses includes no positive clauses, can it be unsatisfiable? Justify your answer. [3 marks]

\[\text{(ii)}\] Use resolution with negative selection to derive a contradiction from the clauses above. [7 marks]