3 Computation Theory (AMP)

(a) Explain how to code register machine programs \( P \) as numbers \( \lceil P \rceil \in \mathbb{N} \) so that each \( e \in \mathbb{N} \) can be decoded to a unique register machine program \( \text{prog}(e) \).

[10 marks]

(b) Find a number \( e_1 \in \mathbb{N} \) for which \( \text{prog}(e_1) \) is a register machine program for computing the function \( \text{one} \in \mathbb{N} \to \mathbb{N} \) with \( \text{one}(x) = 1 \) for all \( x \in \mathbb{N} \).

[2 marks]

(c) Why is it important for the theory of computation that the functions involved in the coding and decoding given in part (a) are themselves register machine computable? (You are not required to prove that they are computable.)

[2 marks]

(d) Define what it means for a set of numbers \( S \subseteq \mathbb{N} \) to be register machine decidable.

[2 marks]

(e) Let \( \varphi_e \in \mathbb{N} \to \mathbb{N} \) denote the partial function of one argument computed by the register machine with program \( \text{prog}(e) \). Prove that \( \{ e \in \mathbb{N} \mid \varphi_e = \text{one} \} \) is register machine undecidable (where \( \text{one} \) is the function mentioned in part (b)). State carefully any standard results that you use in your proof.

[4 marks]