7 Discrete Mathematics (MPF)

(*a*) Let m be a fixed positive integer.

(*i*) For an integer c, let $K_c = \{ k \in \mathbb{N} \mid k \equiv c \pmod{m} \}$.

Show that, for all $c \in \mathbb{Z}$, the set K_c is non-empty. [2 marks]

(*ii*) For an integer c, let κ_c be the least element of K_c.

Prove that for all $a, b \in \mathbb{Z}$, $a \equiv b \pmod{m}$ iff $\kappa_a = \kappa_b$. [4 marks]

(*b*) (*i*) State Fermat's Little Theorem. [2 marks]

(*ii*) Prove that for all natural numbers m and n, and for all prime numbers p, if $m \equiv n \big(\mathrm{mod}\ (p-1)\big)$ then $\forall\, k \in \mathbb{N}.\, k^m \equiv k^n \pmod{p}$. [6 marks]

(*c*) (*i*) Use Euclid's Algorithm to express the number 1 as an integer linear combination of the numbers 34 and 21. [3 marks]

(*ii*) Find a solution $x \in \mathbb{N}$ to $34 \cdot x \equiv 3 \pmod{21}$. [3 marks]