9 Algorithms (TMS)

(a) Explain the terms *amortized analysis, aggregate analysis* and *potential method.*

(b) Consider an arbitrary sequence of $n$ stack operations \texttt{PUSH()}, \texttt{POP()} and \texttt{MULTIPOP(x)} in which \texttt{POP()} or \texttt{MULTIPOP(x)} never attempt to remove more elements than there are on the stack. Assuming that the stack begins with $s_0$ items and finishes with $s_n$ items, determine the worst-case total cost for executing the $n$ operations as a function of $n$, $s_0$ and $s_n$. You may assume \texttt{PUSH()} and \texttt{POP()} cost 1 each and \texttt{MULTIPOP(x)} costs $x$.

(c) Suppose we want to store a number of items in an array, but we do not know in advance how many items need to be stored. The \texttt{INSERT(x)} operation appends an item $x$ to the array. More precisely, if the size of the array is large enough, $x$ is inserted directly at the end of the array. Otherwise, a new array of larger size is created that contains all previous items with $x$ being appended at the end. The total cost of \texttt{INSERT(x)} is 1 in the first case, and the size of the new array in the second case.

(i) Devise a strategy which, for any integer $n$, performs any sequence of $n$ \texttt{INSERT(.)} operations at a total cost of $O(n)$.

(ii) For the strategy described in (c)(i), give a proof of the cost of the algorithm using the potential method.