6 Numerical Methods (DJG)

(*a*) A lander for the planet Mars has initial mass $M_0 = m(0)$ kilograms which includes $F_0 = f(0)$ kilograms of fuel. It is released from an orbiter at time zero at height $H_0 = h(0)$ with an initial downwards velocity of zero. It must touch down at less than 1 metre per second. Its downward force is Mg (where g is constant) and this is countered by a rocket motor that is pre-programmed to generate a time-varying upwards force of $u(t)$. The motor burns fuel at a mass rate proportional to the force it develops. This is summarised in these equations:

$$\frac{dm(t)}{dt} = -\alpha u(t) \qquad \frac{dv(t)}{dt} = \frac{u(t) - gm(t)}{m(t)} \qquad \frac{dh(t)}{dt} = v(t)$$

A discrete-time computer simulation of the landing uses time steps Δt. Using a programming language of your choice or pseudo code:

(*i*) Give a suitable state vector for the system. Include setup code that suitably initialises the state vector. [1 mark]

(*ii*) Give state update assignments for one time step based on simple linear projections assuming a function `u(t)` has been provided. [2 marks]

(*iii*) Give code for the various stopping conditions. These include a safe landing, a fatal crash or running out of fuel. [1 mark]

(*iv*) Why does a simple linear projection lead to a velocity modelling error in every time step. What determines the error magnitude and does it compound over successive steps? [3 marks]

(*b*) (*i*) Briefly describe the bisection method (binary chop) for finding a root of an equation. Mention two possible stopping conditions. [3 marks]

(*ii*) Recall that the CORDIC algorithm uses successive approximation where the ith division of the interval is by $\arctan(2^{-i})$. Give a stopping condition for CORDIC. [2 marks]

(*iii*) The following approximation can be used for cosine: $\cos(x) \approx 1 - \frac{x^2}{2}$. Does it accurately deliver three significant decimal digits where the argument range is 0.0 to $\pi/4$? [2 marks]

(*iv*) Approximately how many iterations of CORDIC are required to ensure three significant decimal digits are accurate over the same range? [6 marks]