5 Numerical Methods (DJG)

(a) Floating point representation:

(i) If a single-precision floating point number is added to a double-precision floating point number, what can we say about the expected and worst-case representation errors in the result? [2 marks]

(b) Floating point rounding:

(i) Would a round-to-odd rule introduce a different amount of bias compared with the normally used round-to-even rule? [2 marks]

(ii) What is it about counting in base 10 that causes bias in the standard approach used when rounding to a given number of significant figures? Explain whether a similar rule for numbers expressed in base 4 would introduce a bias. [4 marks]

(c) Matrix multiplication conditioning:

(i) When a pair of matrices of dimension $N \times N$ are multiplied, what is the expected and worst case representation error in the result? [2 marks]

(ii) Before using Gaussian Elimination to achieve triangle form when solving a system of simultaneous equations, what step or steps can be taken to ensure numerical stability? [4 marks]

(iii) What is meant by backwards stability regarding a numerical method? [2 marks]

(iv) What role can partial derivatives play in examining the stability of a computation compared with using a condition number? [4 marks]