COMPUTER SCIENCE TRIPOS Part IA

Tuesday 3 June 2014 1.30 to 4.30 pm

COMPUTER SCIENCE Paper 2

*Answer **one** question from each of Sections A, B and C, and **two** questions from Section D.*

*Submit the answers in five **separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Blue cover sheets
Tags
Rough work pad

SPECIAL REQUIREMENTS
Approved calculator permitted

SECTION A

1 Digital Electronics

(*a*) A combinational logic circuit takes a 4-bit unsigned binary integer number at its inputs labelled D_3, D_2, D_1 and D_0, where D_3 is the most significant bit. For decimal input 1, 2, 3, 5, 7, 11 and 13, the output S is to be at logic 1, and it is to be at logic 0 otherwise.

(*i*) Write down the truth table for the required combinational logic function.

(*ii*) Using a Karnaugh map, determine the simplified Boolean expression for the output S in terms of the inputs D_3 to D_0 in a minimum sum-of-products form.

(*iii*) Describe what is meant by an essential term in a Karnaugh map. Write down the essential terms for the Karnaugh map in (*ii*).

(*iv*) Using a Karnaugh map, this time determine the required simplified Boolean expression for the output S in a minimum product-of-sums form.

[10 marks]

(*b*) Provide a circuit diagram which implements the following Boolean function using only NAND gates

$$F = (A + \overline{D}).(B + C + \overline{D}).(A + \overline{B} + \overline{C})$$

that has the don't care states: $A.B.\overline{C}.\overline{D}$, $A.B.\overline{C}.D$, $A.B.C.D$ and $A.B.C.\overline{D}$

[4 marks]

(*c*) Show that

$$\begin{aligned}(X + Y).(X + Z) &= X + Y.Z \\ (X + Y).(\overline{X} + Z) &= X.Z + \overline{X}.Y\end{aligned}$$

Using these results or otherwise, simplify the following expression

$$P = (A + B + \overline{C}).(A + B + D).(A + B + E).(A + \overline{D} + E).(\overline{A} + C)$$

[6 marks]

2 Digital Electronics

(*a*) Show how two NOR gates may be connected to form an RS latch. Describe its operation and give a table relating its inputs to its outputs. How could you use this circuit to eliminate the effect of contact bounce in a single pole double throw switch supplying an input to a digital logic circuit? [6 marks]

(*b*) The state sequence for a particular 4-bit binary up-counter is as follows:

Q_A	Q_B	Q_C	Q_D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
1	0	1	1
0	0	0	0
0	0	0	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Show how four negative edge triggered T-type flip-flops (FFs) with outputs labelled Q_A, Q_B, Q_C and Q_D can be used to implement a ripple counter having the specified state sequence. Show any combinational logic necessary assuming that the FFs have asynchronous reset inputs available. [4 marks]

(*c*) Using the principles of synchronous design, determine the next state combinational logic expressions required to implement a counter having the state sequence specified in part (*b*). Assume that D-type FFs are to be used and that unused states do not occur. [4 marks]

(*d*) Explain carefully what happens if the counter in (*c*) starts in state 1 1 1 0 . In general, how can start-up problems be overcome in the design of synchronous state machines? [4 marks]

(*e*) What are the advantages and disadvantages of the synchronous design in part (*c*) compared with the alternative design in part (*b*)? [2 marks]

SECTION B

3 Operating Systems

One goal of a multiuser operating system is to protect each user's information and activity from damage caused by accidental or deliberate actions of other users of the system.

(*a*) Describe a mechanism that operating systems use to reduce the opportunity for a user process to prevent another user's process from making progress. In your description include any particular hardware features that are relied upon. [3 marks]

(*b*) Describe two alternative mechanisms that operating systems could use to reduce the opportunity for a user process to access or corrupt the information being used by another user's process. In your descriptions include any particular hardware features that are relied upon. [6 marks]

(*c*) Describe how an operating system might attempt to ensure that long-term user information (that is, information which exists beyond process execution) is not interfered with or misused by other users. Your description should be clear about when actions are performed and the resources they consume. [5 marks]

(*d*) To what extent are the mechanisms described above useful in single user systems? [3 marks]

(*e*) How do operating systems ensure that they are not themselves overly-restricted by these mechanisms? [3 marks]

4 Operating Systems

(*a*) Describe the difference between *blocking* and *nonblocking* input/output operations. How can an operating system improve the performance (as seen by a process) of blocking operations? [4 marks]

(*b*) A privileged process is given raw access to a slow disk device. It reads a page from the disk (using a blocking operation), processes the information and repeats. Suppose a read takes 3 units of time and the processing 2 units of time, so that reading a block and processing takes 5 units of elapsed time. Assuming the machine is otherwise idle, how can this elasped time be reduced? State any assumptions about hardware features you are making. [5 marks]

(*c*) Describe how polled I/O works and state its disadvantages. Under what conditions is polling a sensible approach? Describe an alternative approach. (You may find it helpful to provide a few lines of psuedo code.) [4 marks]

(*d*) What advantages does direct memory access (DMA) provide? Describe its operation as seen by a device driver in the operating system. (You may find it helpful to write a few lines of psuedo code.) [5 marks]

(*e*) To what extent does heterogeneity in I/O systems add complexity to an operating system? [2 marks]

SECTION C

5 Software and Interface Design

(*a*) Define briefly, for each of the following techniques, what its purpose is and how it is conducted.

(*i*) Regression testing

(*ii*) A/B testing

(*iii*) Unit testing

(*iv*) Load testing

[12 marks]

(*b*) Although each of these techniques can provide new information of value to a software project, costs can be reduced if information is available earlier in the design cycle. For each of the four techniques in part (*a*), suggest a method by which some of the resulting information could be obtained earlier in the project.

[8 marks]

6 Software and Interface Design

The following is an extract from a design brief written by the client for one of the 2014 Cambridge group design projects.

What I'd like is some sort of database of recipes to which I can send queries such as "Find me something that doesn't contain cabbage or tomatoes that takes less than 30 minutes to prepare", or "I've got kohlrabi in the veg box AGAIN, are there any recipes I haven't tried that might make something edible out of it?", or "I've actually got a couple of hours free to cook this weekend, what was that complicated Ottolenghi recipe I flagged two weeks ago to try later?". The database needs to cope with the fact that ingredients can have different names but mean the same thing: e.g. "flour" and "plain flour", and that "1/4 lb" and "4oz" are the same thing and equal to "100g" (and not 113g). It would be great if once I've chosen this week's menu, it could produce a shopping list I can plug into `www.myfavouritesupermarket.com`*, and it needs to be usable by non-engineers.*

(*a*) For each of the following software project phases, suggest a design model or representation that would be a helpful aid in the design process. For each of these, sketch an example to show what this model looks like, based on some part of the above design brief.

(*i*) Inception phase

(*ii*) Elaboration phase

(*iii*) Construction phase

(*iv*) Transition phase

[12 marks]

(*b*) For each of the sketched examples in part (*a*), describe how the design work so far could be evaluated before proceeding to the next phase. [4 marks]

(*c*) Choose two of the above design models, representations or evaluation methods, and explain how they would be done differently if the design project was following an agile rather than spiral project management approach. [4 marks]

SECTION D

7 Discrete Mathematics

(*a*) Let m be a fixed positive integer.

(*i*) For an integer c, let $K_c = \{ k \in \mathbb{N} \mid k \equiv c \pmod{m} \}$.

Show that, for all $c \in \mathbb{Z}$, the set K_c is non-empty. [2 marks]

(*ii*) For an integer c, let κ_c be the least element of K_c.

Prove that for all $a, b \in \mathbb{Z}$, $a \equiv b \pmod{m}$ iff $\kappa_a = \kappa_b$. [4 marks]

(*b*) (*i*) State Fermat's Little Theorem. [2 marks]

(*ii*) Prove that for all natural numbers m and n, and for all prime numbers p, if $m \equiv n \big(\mathrm{mod}\ (p-1)\big)$ then $\forall\, k \in \mathbb{N}.\, k^m \equiv k^n \pmod{p}$. [6 marks]

(*c*) (*i*) Use Euclid's Algorithm to express the number 1 as an integer linear combination of the numbers 34 and 21. [3 marks]

(*ii*) Find a solution $x \in \mathbb{N}$ to $34 \cdot x \equiv 3 \pmod{21}$. [3 marks]

8 Discrete Mathematics

(*a*) Let $\#X$ denote the cardinality of a set X.

Define a unary predicate P for which the statement

$$\forall \text{ sets } X.\ \big[\, P(\#X) \iff (\forall \text{ sets } A, B.\ A \times X = B \times X \implies A = B\,)\big]$$

holds. [1 mark]

Prove the statement for your given predicate P. [4 marks]

(*b*) For sets X, Y, Z, let $(X \Rightarrow Y)$ denote the set of functions from X to Y and let $\mathcal{P}(Z)$ be the powerset of Z.

For sets A and B, consider the function

$$(\cdot)^\sharp : \big(A \Rightarrow \mathcal{P}(B)\big) \longrightarrow \big(\mathcal{P}(A) \Rightarrow \mathcal{P}(B)\big)$$

given, for all $f \in \big(A \Rightarrow \mathcal{P}(B)\big)$ and $X \in \mathcal{P}(A)$, by

$$f^\sharp(X) = \bigcup_{a \in X} f(a)$$

Show that for all $g \in \big(\mathcal{P}(A) \Rightarrow \mathcal{P}(B)\big)$ there exists $f \in \big(A \Rightarrow \mathcal{P}(B)\big)$ such that $f^\sharp = g$ iff, for all $\mathcal{F} \subseteq \mathcal{P}(A)$, $g\big(\bigcup_{X \in \mathcal{F}} X\big) = \bigcup_{X \in \mathcal{F}} g(X)$. [6 marks]

(*c*) For sets S and A, let $\mathrm{Bij}(S, S)$ be the set of bijections from S to S, let $\mathrm{Inj}(S, A)$ be the set of injections from S to A, and let $\mathcal{P}_S(A) = \big\{ X \subseteq A \mid X \cong S \big\}$ be the set of subsets of A that are in bijection with S.

(*i*) Prove that the relation $\approx\ \subseteq \mathrm{Inj}(S, A) \times \mathrm{Inj}(S, A)$ defined, for all $f, g \in \mathrm{Inj}(S, A)$, by

$$f \approx g \iff \exists\, h \in \mathrm{Bij}(S, S)\,.\ f = g \circ h$$

is an equivalence relation. [3 marks]

(*ii*) Define a bijection

$$\mathrm{Inj}(S, A)_{/\approx} \longrightarrow \mathcal{P}_S(A)$$

You need not prove your function is bijective, but you should explain why your mapping is well defined. [6 marks]

9 Discrete Mathematics

(*a*) The subset S of $\mathbb{N} = \{0, 1, 2, \ldots\}$ is inductively defined by the following axiom and rules, where n ranges over $\mathbb{N}$:

$$\frac{}{1} \qquad \frac{n}{2n} \qquad \frac{n}{3n} \qquad \frac{n+5}{n}$$

(*i*) State the principle of Rule Induction associated with this set of axioms and rules. [4 marks]

(*ii*) Use Rule Induction to prove that no element of S is divisible by 5. [4 marks]

(*iii*) Is 0 an element of S? Justify your answer. [1 mark]

(*b*) State the principle of Mathematical Induction. [2 marks]

(*c*) For sets X and Y of strings over an alphabet Σ, let XY denote the set $\{uv \mid u \in X \text{ and } v \in Y\}$ of all concatenations of a string in X followed by a string in Y. For $n \in \mathbb{N}$, let X^n be given by: $X^0 = \{\varepsilon\}$ (where ε denotes the null string) and $X^{n+1} = XX^n$. Let $X^* = \bigcup_{n \geq 0} X^n$.

Suppose $X, Y, Z \subseteq \Sigma^*$ satisfy $Z = XZ \cup Y$.

(*i*) Prove by Mathematical Induction that $\forall n \in \mathbb{N}.\, X^n Y \subseteq Z$ and deduce that $X^* Y \subseteq Z$. [4 marks]

(*ii*) Suppose further that $\varepsilon \notin X$. By considering the property of $n \in \mathbb{N}$ given by $\forall w \in Z.\, |w| \leq n \Rightarrow w \in X^* Y$, or otherwise, use Mathematical Induction to prove that $Z \subseteq X^* Y$. [5 marks]

10 Discrete Mathematics

(*a*) For each symbol x in the alphabet $\Sigma = \{a, b, c\}$, let O_x be the language over Σ consisting of all strings that contain an odd number of occurrences of the symbol x; and let E_x be the language of strings over Σ containing an even number of occurrences of the symbol x.

(*i*) Give a deterministic finite automaton whose language of accepted strings is O_a. [2 marks]

(*ii*) Give a regular expression whose language of matching strings is O_a. [2 marks]

(*iii*) Give a deterministic finite automaton whose language of accepted strings is $O_a \cap E_b$. [4 marks]

(*b*) $M = (Q, \Sigma, \delta, s, F)$ is a deterministic finite automaton whose set of states Q has ℓ elements. Suppose that M accepts a string $w \in \Sigma^*$ whose length $|w|$ satisfies $|w| \geq \ell$.

(*i*) Show that $w = u_1 v u_2$ for some strings $u_1, v, u_2 \in \Sigma^*$ such that $|u_1| < \ell$, $1 \leq |v| \leq \ell$ and M accepts $u_1 v^n u_2$ for all $n \in \mathbb{N} = \{0, 1, 2, \ldots\}$. [7 marks]

(*ii*) Hence show that if infinitely many strings are accepted by M, then it must accept some string w' with $\ell \leq |w'| < 2\ell$. [5 marks]

END OF PAPER