4 Denotational Semantics (MPF)

(a) (i) State carefully, without proof, the compositionality, soundness, and adequacy results for PCF. [6 marks]

(ii) Define the notion of contextual equivalence in PCF. [2 marks]

(You need not describe the syntax and the operational and denotational semantics of PCF.)

(b) Show that for all types $\tau$ and closed terms $M$ and $M'$ of type $\tau$, if $[M]$ and $[M']$ are equal elements of the domain $[\tau]$ then $M$ and $M'$ are contextually equivalent. [4 marks]

(c) Consider the following closed PCF terms of type $nat \to bool \to nat$:

$F_0 = \text{fn } x : nat. \text{fn } y : bool. x$

$F_1 = \text{fix( fn } f : nat \to bool \to nat. \text{fn } x : nat. \text{fn } y : bool.\text{ if zero}(x) \text{ then 0 else succ}( f (\text{pred } x) y ) )$

$F_2 = \text{fn } x : nat. \text{fn } y : bool. \text{if } y \text{ then } x \text{ else } x$

State whether or not $F_1$ and $F_2$ are contextually equivalent to $F_0$. Justify your answers. [4 marks each]