4   Denotational Semantics (MPF)

(a) (i) State carefully, without proof, the compositionality, soundness, and adequacy results for PCF.   [6 marks]

(ii) Define the notion of contextual equivalence in PCF.   [2 marks]

(You need not describe the syntax and the operational and denotational semantics of PCF.)

(b) Show that for all types τ and closed terms M and M' of type τ, if $[\![M]\!]$ and $[\![M']\!]$ are equal elements of the domain $[\![\tau]\!]$ then M and M' are contextually equivalent.   [4 marks]

(c) Consider the following closed PCF terms of type $nat \to bool \to nat$:

$$
\begin{aligned}
F_0 \;&=\; \textbf{fn } x : nat.\ \textbf{fn } y : bool.\ x \\[4pt]
F_1 \;&=\; \textbf{fix}\big(\ \textbf{fn } f : nat \to bool \to nat.\ \textbf{fn } x : nat.\ \textbf{fn } y : bool. \\
&\qquad\qquad \textbf{if zero}(x)\ \textbf{then } \mathbf{0} \\
&\qquad\qquad \textbf{else succ}(\ f\,(\textbf{pred } x)\,y\,)\ \big) \\[4pt]
F_2 \;&=\; \textbf{fn } x : nat.\ \textbf{fn } y : bool.\ \textbf{if } y\ \textbf{then } x\ \textbf{else } x
\end{aligned}
$$

State whether or not F_1 and F_2 are contextually equivalent to F_0. Justify your answers.   [4 marks each]

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