13 Topics in Concurrency (GW)

This question is on HOPLA and PCCS, a variant of pure CCS in which any output on a channel persists. Let $A$ be a set of channel names ranged over by $a, b, c$ and let $\bar{A}$ be the set of complemented channel names, $\bar{A} = \{ \bar{a} \mid a \in A \}$. The set of labels $L = A \cup \bar{A}$ is ranged over by $l$, to which we extend complementation by taking $\bar{\bar{l}} = l$.

Use $\alpha$ to range over $L \cup \{ \tau \}$, where $\tau$ is a distinct label. The terms of PCCS follow the grammar $P ::= \text{nil} | \bar{a} | a.P | (P_1 \parallel P_2)$. The operational semantics of PCCS is:

\[
\begin{align*}
\bar{a} &\xrightarrow{\alpha} \bar{a} \\
\bar{a}P &\xrightarrow{\alpha} P \\
\text{nil}P &\xrightarrow{\alpha} P \\
\bar{P}_1 &\xrightarrow{\alpha} P'_1 \\
P_1 &\parallel P_2 \xrightarrow{\alpha} P'_1 \parallel P_2 \\
P_1 &\parallel \bar{P}_2 \xrightarrow{\alpha} P_1 \parallel P'_2 \\
\bar{P}_1 &\parallel \bar{P}_2 \xrightarrow{\alpha} P'_1 \parallel P'_2
\end{align*}
\]

(a) Draw the transition system of the PCCS term $\bar{a} \parallel a.a.\bar{b}$. [3 marks]

(b) This part of the question is on HOPLA. For reference, the operational semantics of HOPLA is presented at the end of the question.

(i) For $u$ of sum type, let $[u > a.x \Rightarrow t]$ abbreviate $[\pi_a(u) > .x \Rightarrow t]$. Derive a rule for the transitions of $[u > a.x \Rightarrow t]$. [2 marks]

(ii) Show that $[a.u > a.x \Rightarrow t] \sim [t[u/x]]$ and $[a.u > b.x \Rightarrow t] \sim \text{nil}$ if $a \neq b$, where $\text{nil}$ represents the empty sum and $\sim$ is the bisimilarity of HOPLA. [4 marks]

(c) Write down a HOPLA term realising the parallel composition of PCCS. Use this to give an encoding of PCCS into HOPLA, specifying a HOPLA term $[P]$ for every PCCS term $P$. [Hint: The realisation of parallel composition should be the same as that of the encoding of pure CCS into HOPLA.] [5 marks]

(d) Use the rules of HOPLA to show how a derivation establishing $[P_1 \parallel P_2] \xrightarrow{\alpha} [P'_1 \parallel P_2]$ can be constructed from a derivation of $[P_1] \xrightarrow{\alpha} [P'_1]$. Explain briefly how you would show that if $P \xrightarrow{\alpha} P'$ in PCCS then $[P] \xrightarrow{\alpha} [P']$ in HOPLA. In what part of the proof would the derivation that you have constructed be useful? [6 marks]

Subject to suitable typings, HOPLA has transitions $t \xrightarrow{p} t'$ between closed terms $t, t'$ and action $p$ given by the following rules:

\[
\begin{align*}
\text{rec } x. t &\xrightarrow{p} t' \\
\text{rec } x. t &\xrightarrow{p} t' \\
\sum_{i \in I} t_i &\xrightarrow{p} t' (j \in I) \\
\text{nil} &\xrightarrow{p} t \\
u \Rightarrow u' &\xrightarrow{t[u'/x]} t' \quad [u > .x \Rightarrow t] \xrightarrow{p} t'
\end{align*}
\]
\[
\begin{align*}
\frac{t[u/x] \xrightarrow{P} t'}{\lambda x t \xrightarrow{u \rightarrow P} t'} & \quad \frac{t \xrightarrow{u \rightarrow P} t'}{t u \xrightarrow{P} t'} & \quad \frac{t \xrightarrow{P} t'}{\alpha t \xrightarrow{\alpha P} t'} & \quad \frac{t \xrightarrow{\alpha P} t'}{\pi_a(t) \xrightarrow{P} t'}
\end{align*}
\]