9 Information Theory and Coding (JGD)

(a) Consider an alphabet of 5 symbols whose probabilities are as follows:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>1/16</td>
<td>1/3</td>
<td>1/8</td>
<td>1/16</td>
<td>1/2</td>
</tr>
</tbody>
</table>

One of these symbols has been selected at random and you need to discover which symbol it is by asking ‘yes/no’ questions that will be truthfully answered.

(i) What would be the most efficient sequence of such questions that you could ask in order to discover the selected symbol? [2 marks]

(ii) By what principle can you claim that each of your proposed questions in the sequence is maximally informative? [2 marks]

(iii) On average, how many such questions will need to be asked before the symbol is discovered? What is the entropy of the symbol set? [2 marks]

(iv) Construct a uniquely decodable prefix code for the symbols. Explain why it is uniquely decodable and why it has the prefix property. [2 marks]

(v) Relate the bits in the code words forming your prefix code to the ‘yes/no’ questions that you proposed in (i). [2 marks]

(b) Explain how the bits in an IrisCode are set by phase sequencing. Discuss how quantisation of the complex plane into phase quadrants sets each pair of bits; why it is beneficial for quadrant codes to form a Gray Code; how much entropy is thereby typically extracted from iris images; and why such bit sequences enable extremely efficient identity searches and matching. [5 marks]

(c) Consider a noisy analog communication channel of bandwidth $\omega = 1$ MHz, which is perturbed by additive white Gaussian noise whose total spectral power is $N_0\omega = 1$. Continuous signals are transmitted across such a channel, with average transmitted power $P = 1,000$. Give a numerical estimate for the channel capacity, in bits per second, of this noisy channel. Then, for a channel having the same bandwidth $\omega$ but whose signal-to-noise ratio $\frac{P}{N_0\omega}$ is four times better, repeat your numerical estimate of capacity in bits per second. [5 marks]