This question is about a language that is like L1 but with a stack instead of a store.

(a) Consider the following grammars for expressions e and values v:

\[
\begin{align*}
e & ::= \text{push}(e) \mid \text{pop}() \mid \text{skip} \mid e_1 ; e_2 \mid \text{true} \mid \text{false} \mid \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \\
v & ::= \text{skip} \mid \text{true} \mid \text{false}.
\end{align*}
\]

The configurations for this language are pairs \(\langle e, bs \rangle\) where \(e\) is an expression and \(bs\) is a finite list of booleans.

The operational semantics of \(\text{push}(e)\) and \(\text{pop}()\) are defined by the following rules:

\[
\begin{align*}
\langle \text{push}(\text{true}) , bs \rangle & \rightarrow \langle \text{skip} , (\text{true} :: bs) \rangle \\
\langle e , bs \rangle & \rightarrow \langle e' , bs' \rangle \\
\langle \text{push}(\text{false}) , bs \rangle & \rightarrow \langle \text{skip} , (\text{false} :: bs) \rangle \\
\langle \text{pop}() , b :: bs \rangle & \rightarrow \langle b , bs \rangle
\end{align*}
\]

Write down rules for the other language constructs, to define a reasonable operational semantics. [5 marks]

(b) The types for this language are

\[
T ::= \text{unit} \mid \text{bool}
\]

We define a relation \(e : T\) between expressions and types. The types of \(\text{push}(e)\) and \(\text{pop}()\) are given by the following rules:

\[
\begin{align*}
e : \text{bool} & \hspace{1cm} \text{push}(e) : \text{unit} \\
\text{pop}() : \text{bool}
\end{align*}
\]

Write down rules for the other language constructs to define a reasonable type system. [5 marks]

(c) Consider the following statements:

(i) For all pairs of configurations \(\langle e , bs \rangle, \langle e' , bs' \rangle\), and all types \(T\):

if \(e : T\) and \(\langle e , bs \rangle \rightarrow \langle e' , bs' \rangle\) then \(e' : T\).

(ii) For all configurations \(\langle e , bs \rangle\) and all types \(T\):

if \(e : T\) then either \(e\) is a value or there is a configuration \(\langle e' , bs' \rangle\) such that \(\langle e , bs \rangle \rightarrow \langle e' , bs' \rangle\).

For each of these two statements, state whether it holds. If it holds, prove it. If it doesn’t hold, explain why and suggest a change to the semantics that would make the theorem hold. [10 marks]