3 Computation Theory (AMP)

(a) What does it mean for a register machine to be universal? [4 marks]

(b) Define what it means for a partial function \( f \in \mathbb{N}^n \rightarrow \mathbb{N} \) to be register machine computable. [3 marks]

(c) Show that the following functions \( f, g, h, k \) are register machine computable.

\( (i) \) The partial function \( f \in \mathbb{N} \rightarrow \mathbb{N} \) that is everywhere undefined. [1 mark]

\( (ii) \) \( g(x_1, x_2) = \begin{cases} x_1 - x_2 & \text{if } x_1 \geq x_2 \\ 0 & \text{if } x_1 < x_2 \end{cases} \) [4 marks]

\( (iii) \) \( h(x_1) = \begin{cases} 2^{x_1-1} & \text{if } x_1 > 0 \\ \text{undefined} & \text{if } x_1 = 0 \end{cases} \) [4 marks]

\( (iv) \) \( k(x_1, x_2) = 1 \) if the register machine program with index \( x_1 \), when started with 0 in all registers, halts in at most \( x_2 \) steps; and \( k(x_1, x_2) = 0 \) otherwise. [4 marks]